

# 1 Lesson 3: Screen and Document Appearance, More on Graphing

## 1.1 Tag Appearance

Tag Appearance, like the scale factor, deals with the way your file appears on your computer screen. This has **nothing** to do with the appearance of your typeset document.

### 1.1.1 Marginal Cost

Several concepts in economics that have to do with rates of change can be effectively described with the methods of calculus. Among these are marginal cost and marginal revenue.

If the *cost function*  $C(x)$  is the cost of producing  $x$  units of a certain product, then the *marginal cost* is the rate of change of  $C$  with respect to  $x$ . In other words, the marginal cost function is the derivative,  $C'(x)$ , of the cost function.

The *average cost function*

$$c(x) = \frac{C(x)}{x}$$

represents the cost per unit when  $x$  units are produced. Now  $c'(x) = 0$  when  $x C'(x) - C(x) = 0$  and this gives

$$C'(x) = \frac{C(x)}{x} = c(x)$$

Therefore, if the average cost is a minimum, then the marginal cost equals the average cost.

**Example 1** A company estimates that the cost in dollars of producing  $x$  items is

$$C(x) = 2600 + 2x + .001x^2$$

Find the cost, average cost, and marginal cost of producing 1000 items. At what production level will the average cost be lowest, and what is this minimum average cost?

**Solution 2** The average cost function is

$$c(x) = \frac{C(x)}{x} = \frac{2600}{x} + 2 + 0.001x$$

and the marginal cost function is

$$C'(x) = 2 + 0.002x$$

Consequently, when producing 1000 the cost is  $C(1000) = \$5600.00$ , the average cost is  $c(1000) = \$5.60$ , and the marginal cost is  $C'(1000) = \$4.00$ .

The production level that minimizes the average cost occurs when the marginal cost equals the average cost. That is, when

$$2 + 0.002x = \frac{2600}{x} + 2 + 0.001x$$

which has the two solutions  $-1612.5, 1612.5$ . We see that the average cost is minimized when 1612 units are produced. Note that

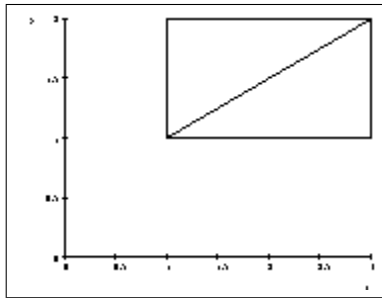
$$c''(1612.5) = 1.2402 \times 10^{-6} > 0.$$

The minimum average cost is  $(c(1612) = 5.2249)$  \$5.22/item

## 1.2 Plotting squares and triangles

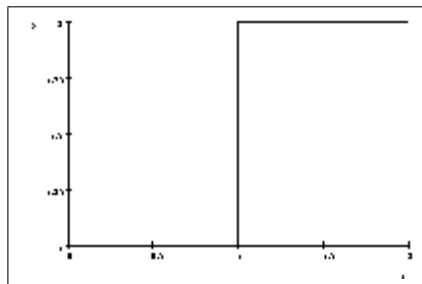
One way to accomplish this is to place the coordinates of, say, a triangle in an array as follows

(1, 1, 3, 1, 3, 1, 3, 2, 3, 2, 1, 1)



Now we will graph a rectangle. First, form a 4 by 2 matrix and use as row entries the vertices of the rectangle:

1 1  
3 1  
3 2  
1 2  
1 1



1.2	1.2
2.8	1.2
2.8	1.8
1.2	1.2

## 2 Project 3

Instructions: Create a file containing the items contained in this document. Submit .tex and .pdf versions of your file to [teprice@uakron.edu](mailto:teprice@uakron.edu). The name of your files should be of the form **yourlastname03.tex** and **yourlastname03.pdf**. All calculations should be done using the CAS in SWP.

### 2.1 Marginal Revenue

Let  $p(x)$  be the price per unit that the company can charge if it sells  $x$  units. Then  $p$  is called the demand function (or *price function*) and we would expect it to be a decreasing function of  $x$ . If  $x$  units are sold and the price per unit is  $p(x)$ , then the total revenue is

$$R(x) = xp(x)$$

and  $R$  is called the revenue function (or *sales function*). The derivative  $R'$  of the revenue function is called the *marginal revenue function* and is the rate of change of revenue with respect to the number of units sold.

If  $x$  units are sold, then the total profit is

$$P(x) = R(x) - C(x)$$

and  $P$  is called the profit function. The marginal profit function is  $P'$ , the derivative of the profit function. In order to maximize profit we look for the critical numbers of  $P$ , that is, the numbers where the marginal profit is 0. But if  $P''(x) = R''(x) - C''(x) = 0$  then  $R'(x) = C'(x)$ . Therefore, if the profit is a maximum, then the marginal revenue equals the marginal cost.

To ensure that this condition gives a maximum you could use the Second Derivative Test. Note that  $P''(x) = R''(x) - C''(x) < 0$  when  $R''(x) < C''(x)$ , and this condition says that the rate of increase of marginal revenue is less than the rate of increase of marginal cost. Thus the profit will be a maximum when  $R'(x) = C'(x)$  and  $R''(x) < C''(x)$ .

**Exercise 3** (SPECIAL INSTRUCTIONS: *Provide the solution to this exercise in a solution environment with the lead-in text and background and paragraph tags changed from the default. You may choose your fonts and colors.*) Given the cost function

$$C(x) = 1450 + 36x - x^2 + .001x^3$$

and the demand function

$$p(x) = 60 - .01x$$

find the production level that will maximize profit.

**Exercise 4** Create a graph containing two different colored, polygonal line segments of your choosing.