

# 1 Lesson 2: The Basics of Scientific Workplace

## 1.1 An introduction to environments.

**Exercise 1** *This is an exercise environment. It will continue until I use the enter key. This is an exercise environment. It will continue until I use the enter key. (backspace)*

**Exercise 2** ... type the exercise here ...

**Solution 3** ... type the solution here ...

```
\begin{exercise}
...
type the exercise here
...
\begin{solution}
...
type the solution here
...
\end{solution}
\end{exercise}
```

An alternate approach...

**Exercise 4** ... type the exercise here ...

**Solution 5** ... type the solution here ...

```
\begin{exercise}
...
type the exercise here
...
\end{exercise}
\begin{solution}
...
type the solution here
...
\end{solution}
```

## 1.2 More on displayed equations

**Theorem 6** *Let  $a$  be any real number. Then  $\lim_{x \rightarrow a} \frac{x^2 - a^2}{x - a}$*

$$\lim_{x \rightarrow a} \frac{x^2 - a^2}{x - a} = 2a$$

**Proof.** Consider

$$\begin{aligned}\lim_{x \rightarrow a} \frac{x^2 - a^2}{x - a} &= \lim_{x \rightarrow a} \frac{-(a - x)(a + x)}{x - a} \\ &= \lim_{x \rightarrow a} (a + x) \\ &= 2a.\end{aligned}$$

■

This completes Lesson 2.

## 2 Project 2

Instructions: Create a file containing the items contained in this document. Submit your file to [teprice@uakron.edu](mailto:teprice@uakron.edu). The name of your file should be of the form **yourlastname02.tex**. All calculations should be done using the CAS in SWP.

## 3 Derivatives

Differential calculus is concerned with how one quantity changes in relation to another quantity. The central concept of differential calculus is the derivative.

### 3.1 Derivative

**Definition 7** The derivative of a function  $f$  at a number  $a$ , denoted by  $f'(a)$ , is

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a + h) - f(a)}{h}$$

if this limit exists.

**Exercise 8** Use the above definition to compute  $f'(2)$  if  $f(x) = 3x + 2$ .

**Solution 9** (SPECIAL INSTRUCTIONS: This solution environment should be at

Level 2.) Consider

$$\begin{aligned} f'(2) &= \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} \\ &= \lim_{h \rightarrow 0} \frac{3(2+h) + 2 - 8}{h} \\ &= \lim_{h \rightarrow 0} \frac{3h}{h} \\ &= 3. \end{aligned}$$

### 3.2 Equation of Tangent Line

**Theorem 10** If  $f'(a)$  exists, then an equation of the tangent line to the curve  $y = f(x)$  at the point  $(a, f(a))$  is

$$y - f(a) = f'(a)(x - a)$$

### 3.3 Differentiable Functions

**Definition 11** A function  $f$  is differentiable at  $a$  if  $f'(a)$  exists. It is differentiable on an open interval  $(a, b)$  [or  $(a, \infty)$  or  $(-\infty, a)$  or  $(-\infty, \infty)$ ] if it is differentiable at every number in the interval.

### 3.4 Differentiable Implies Continuous

**Theorem 12** If a function  $f$  is differentiable at  $a$ , then  $f$  is continuous at  $a$ .

**Proof.** (SPECIAL INSTRUCTIONS: You should complete the proof of this theorem. Recall that a function  $f$  is continuous at  $a$  if

$$\lim_{x \rightarrow a} f(x) = f(a).$$

If you have trouble proving this, look for the proof in your calculus book.) ■