Pattern

Stefan Forcey, University of Akron

January 9, 2013
sequence.

1, 1, 2, 3, 5, 8, 13, ?
Fibonacci tree.

1, 1, 2, 3, 5, 8, 13, 21, 34, ...
Logarithmic spiral.
Logarithmic spiral. Image credits: McSweeney’s (first two) Wikimedia (last two).

\[ r = e^{\theta}. \]
Mandelbrot sequences.

**Definition**

*Mandelbrot sequences*: Take a complex number $z$ and add it to its square. Repeat, by squaring your result and adding the same original $z$.

\[ z \mapsto z + z^2 \]
Mandelbrot sequences.

Definition

Mandelbrot sequences: Take a complex number $z$ and add it to its square. Repeat, by squaring your result and adding the same original $z$.

$$
\begin{align*}
z & \mapsto z + z^2 \\
& \mapsto z + (z + z^2)^2
\end{align*}
$$

Stefan Forcey, University of Akron
Example Mandelbrot sequence: \( z \mapsto z + z^2 \)

Starting point: \((.3, .1)\)
Mandelbrot $z \mapsto z + z^2$

Starting point: (.3,.1)
Mandelbrot $z \mapsto z + z^2$

Starting point: (.3,.1)
Mandelbrot $z \mapsto z + z^2$

Starting point: (.3,.1)
Mandelbrot $z \mapsto z + z^2$

Starting point: $(.3,.1)$
Mandelbrot \( z \mapsto z + z^2 \)

Starting point: (.3,.1)
Mandelbrot $z \mapsto z + z^2$

Starting point: (.3,.1)
Mandelbrot $z \mapsto z + z^2$

Starting point: (.3,.1)
Mandelbrot \( z \mapsto z + z^2 \)

Starting point: \((-1.1, .22)\)

Real
Logarithmic spiral.  Image credits: Wikimedia (first), NASA (second two).

\[ \ln r = \theta. \]
Mandelbrot $z \mapsto z + z^2$

Starting point: $-0.24 + 0.635i$
\ln r = \theta.
Mandelbrot $z \mapsto z + z^2$

Starting point: $-0.26 + 0.635i$
Mandelbrot set.
Victoria crater.
Mandelbrot set.
Mandelbrot set.
Mandelbrot set.
Mandelbrot set.
Mandelbrot set.
Mandelbrot set.
Mandelbrot set.
Mandelbrot set.
Trees

Image credits: Wikimedia.

Stefan Forcey, University of Akron
Fibonacci again!

1
3
5
8
13

Stefan Forcey, University of Akron
Trees

Image credits: Google Earth, Wikimedia (you may guess which!).

Stefan Forcey, University of Akron
Trees

Episodic radiations in the fly tree of life, Wiegmann et.al. PNAS 2011
**Spirals**

*Image credit: Wikimedia.*

**Starting point:** $0.3 + 0.03i$
Starting point: $-0.26 + 0.635i$

There are many more patterns to be discovered than there are already known...