

Structured categories

INSTRUCTOR: Dr. Stefan Forcey

TEXT and COVERAGE:

Mac Lane: Categories for the working mathematician.

Plan:

- I. Categories and Functors
 - A. Categories: Top, Group, posets, group G , Cat
 - B. Functors: forgetful, free, group homomorphisms G to H
 - C. Natural transformations
 - give definition
 - give two examples, with proof that they fit the definition.
 - D. Cat
 - describe the category of categories (with functors the arrows), i.e. show that it fits the definition of a category
 - given two categories, describe the category of functors from the first to the second (with natural transformations the arrows); again show that it fits the definition of a category
 - describe the category of categories with natural transformations as the arrows.
- II. Monoidal categories
 - A. Bifunctors
 - Pg 36 products of categories, definition of bifunctor
 - B. Products
 - Define: strict monoidal category, 2 examples; monoidal category
 - Pay special attention to the pentagon.
 - Read Coherence pg 165
 - Draw similar diagrams for 5 items (exercise 1 pg 170)
 - C. examples: abelian groups, vector spaces, sets, Cat
 - Describe the product for each of these. The first two are tensor products (monoidal) and the second two are Cartesian (strict).
 - D. monoids
 - E. monoidal functors
- III. Braidings and symmetries
 - A. definitions of braided and symmetric monoidal categories
 - B. 2 examples of each
 - C. The braid group on n strands
 - D. Naturality of the braiding
 - E. Coherence for braiding
 - demonstrate commutativity of diagrams in Joyal-Street (p45)
 - F. The free braided category
- IV. Iterated monoidal categories
 - A. definition and examples

- B. 2-fold monoidal categories based on a braiding
 - check that a braided monoidal category gives a 2-fold monoidal category with $\eta = 1 \times c \times 1$, and show that a 2-fold monoidal category with η an isomorphism is braided with $c = \eta_{1,A,B,1}$
- V. 2-categories
 - A. Enrichment
 - B. Inherited structure
- VI. N-categories
 - A. iterated enrichment
 - B. weak enrichment
 - C. Associahedra and weak n-categories