Polytopes, positrons, and antipodes: Advanced Linear algebra and geometric combinatorics with applications.

INSTRUCTOR: Dr. Stefan Forcey

TEXT and COVERAGE:

Hopf Algebras and their Actions on Rings. Susan Montgomery.

Bibliography:

For relevant journal articles, see the reference section at the end of:
http://faculty.tnstate.edu/sforcey/prop_graph_pb.pdf

For more details, see online notes at
http://faculty.tnstate.edu/sforcey/class_home/research.htm

For making polytopes, try
http://www.math.tu-berlin.de/polymake/index.html#apps/webdemo.html

Here are some shapes!
http://www.ams.org/featurecolumn/archive/associahedra.html

and more…
http://www.claymath.org/library/academy/LectureNotes05/Lodaypaper.pdf

On the first page of the above, J.L.Loday mentions convex hulls. You can check out the Wikipedia about that:
http://en.wikipedia.org/wiki/Convex_hull

or my video!
http://www.ehow.com/video_4975015_tips-learning-geometry.html

Plan of Notes: These may be typed in Latex or Word. Sources must be cited for definitions and examples. Definitions must be self-contained, examples must be self-contained as well as elementary as possible!

I. Advanced linear algebra.

Finish by:_________.

A. Define for review: field, vector space, linear transformation, basis.
B. Define: Tensor product of vector spaces. Give an example.
C. Define: Algebra. Give an example.
D. Define: Graded Algebra. Give an example.
E. Define: Tensor product of Algebras. Show that this product is itself an algebra.
F. Define: coalgebra. Give an example.
G. Define: Hopf Algebra. Give an example.
H. Define: Graded Hopf Algebra.
I. Define: Hopf Module. Give an example.
J. Define: Hopf Algebra morphism, Hopf module morphism.

II. Polytopes: combinatorics and geometry

Finish by: _________.

A. Define: polytope, using the convex hull construction. Give an example in 2,3,4 dimensions (use the Schlegel diagram.)
B. Define: polytope, using the intersection of half planes.
C. Define: K(n), the nth associahedron, using binary trees and convex hulls. How are the associahedra related to each other?
D. Define: J(n), the nth multiplihedron. How are the multiplihedra related to each other?
E. Define: P(n), the nth permutohedron. How are the permutohedra related to each other?
F. Define CK(n), the nth composihedron.
G. Define C(n), the nth cube.
H. Define Δ(n), the nth simplex.
I. Optional: Define the cylohedron, graph-associahedron, and graph-multiplihedron.
J. Project: Construct ten different graph and multigraph associahedra. Construct 5 different graph and multigraph multiplihedra. Conjecture on the relationships.
K. Project: measure the lengths of edges and the total volumes of various faces of the convex hull realizations of the

multiplihedra. Also compute centroids. Conjecture on the formula for finding these in general. Repeat for the composihedra.

L. Project: develop generating functions and combinatorial formulas for the face vectors of the multiplihedra, composihedra, etc.

III. Geometric – Combinatorial Hopf Structures

Finish by :__________.

A. Define the Loday-Ronco Hopf algebra of binary trees. Demonstrate each axiom two separate times.
B. Define the Malvenuto-Reutenauer Hopf algebra of permutations. Demonstrate each axiom two separate times.
C. Define the Hopf Algebra of quasisymmetric functions. Demonstrate the axioms.
D. Project: Define a new Hopf algebra structure on the vertices of the multiplihedra. Check axioms to confirm or disprove its validity.
E. Project: Define a new Hopf module structure on the vertices of the multiplihedra. Check axioms to confirm or disprove its validity.
F. Project: Define a new Hopf module structure on the vertices of the composihedra. Check axioms to confirm or disprove its validity.
G. Project: Define a new Hopf module or Hopf algebra structure on the vertices of the cubes. Check axioms to confirm or disprove its validity.
H. Project: Same as D-G, but using vertices of graph associahedra, multiplihedra, or composihedra.

IV. Project: Application to Quantum Electrodynamics

A. Define the noncommutative Connes-Kreimer Hopf algebra of Feynman diagrams.
B. Prove that the Connes-Kreimer algebra is isomorphic to the Loday-Ronco algebra.
C. Find the antipode of four different Feynman diagrams

V. Optional: Higher categorical structures and Hopf algebras.

A. Category Theory review
B. Species, Posets, Operads and PROPS
C. Incidence algebras
D. Operad algebras
E. Project: Define a procedure for turning an operad module into a Hopf module.