**Topology II: Algebraic Topology**

**INSTRUCTOR:** Dr. Stefan Forcey

**TEXT and COVERAGE**:

Munkres, Topology Second edition.
Hatcher, Algebraic Topology: http://www.math.cornell.edu/~hatcher/AT/ATpage.html

**BIBLIOGRAPHY:**

Prasolov, V.V., *Intuitive Topology* A.M.S.
Weeks, Jeffrey. *The Shape of Space*.

Plan:

I. Functors
   A. Categories: Top, Group, posets, group G, Cat
   B. Functors: forgetful, free, group homomorphisms G to H, topological invariants: homeomorphism and isomorphism

II. Fundamental Group
   A. cell complexes and homotopy
      1. homeomorphism classification of surfaces: orientability, genus/ Euler characteristic, boundary components.
      2. CW complexes
      3. homotopic maps
      4. homotopic spaces and deformation retracts
      5. homotopic vs homeomorphic
   B. loop spaces/ equivalence classes of paths
   C. group presentations (proj. 1)
   D. Van Kampen theorem (proj. 2)
   E. Projects:
      1. What can the fundamental group tell us about the Klein bottle, torus, the once and twice punctured tori (with boundary), and the once, twice and thrice punctured sphere (with boundary)?
         a.) Define the 7 spaces (with pictures). Prove that homotopic spaces have isomorphic fundamental groups, i.e. that a homotopy equivalence between spaces is taken to a group isomorphism by the fundamental group functor. Describe how to find generators of pi_1, describe how to find a relation in pi_1: How do we know when we have all the generators and relations?
         b.) Find generators and relations for the 7 fundamental groups in question. Describe several ways we can know whether two groups are isomorphic or not.
         c.) Distinguish up to homotopy the spaces in question. Also distinguish them up to homeomorphism. How are homotopy and homeomorphism classes related?

2. Find the knot group of a closed braid, in terms of a choice of generators of the braid group.

a.) Explain the algorithm for finding a presentation of the knot group for any knot. This is described on p55. Translate the book's description into a description based on labeling the arcs in a knot diagram.

b.) Explain an algorithm that takes as input a braid presentation of a knot in the standard braid generators and outputs a knot group presentation.

c.) Implement the algorithm from b) (by hand or programming) on three closed braid examples. All should be the same knot. Describe how the algorithm turns a Markov move into a group isomorphism.

III. Homology

A. Simplicial and Singular homology
B. Abelianization of fundamental group
C. Exact sequences, Excision
D. Mayer-Vietoris sequences
E. Projects:

1. Distinguish the solid torus from the 3-ball with toroidal hole and the solid torus with a missing 3-ball.

a) calculate using Mayer Vietoris the homology of the solid torus
b) calculate using Mayer Vietoris the homology of the solid 3-ball with toroidal hole
c) calculate using Mayer Vietoris the homology of the solid torus with a missing 3-ball.
d) calculate using Mayer Vietoris the homology of the solid torus with a with toroidal hole.

2. Use Brouwer fixed point theorem to prove there is a Nash equilibria for an iterated prisoners' dilemma or the game of life.

IV. Higher homotopy

A. Excision, relative homotopy
B. Eckmann-Hilton argument

V. Spectra