Discrete. Spring '20 Quiz 5. Name __________________________ Time _______________________

For each proof, fill in the section Assume and Show.

1. Find the terms of the sequence $a_n$ for $n = 1, 2, 3, 4$.
   
   $a_n = (n \mod 3)^2 - 2^{n-1}$.

   $\begin{array}{c|c}
   n & a_n \\
   \hline
   1 & 1 - 1 = 0 \\
   2 & 4 - 2 = 2 \\
   3 & 0 - 4 = -4 \\
   4 & 1 - 8 = -7 \\
   \end{array}$

2. Use a direct proof to prove: $\forall z \in \mathbb{Z}, z \mod 3 = 2 \Rightarrow z^2 \mod 3 = 1$.

   Assume: $z = 3k + 2$ (remainder 2)
   
   Show: $z^2 = 3p + 1$ (remainder 1)
   
   Proof:
   $z^2 = (3k + 2)^2$
   
   $= 9k^2 + 12k + 4$
   
   $= 3(3k^2 + 4k + 1) + 1$
   
3. Use the Principle of Mathematical Induction to prove $\forall n \in \mathbb{Z}, n \geq 2$ implies $5 \mid (7^n - 2^n)$.

   Base case checked: $n = 2$ : $7^2 - 2^2 = 49 - 4 = 45 = 9 \times 5$  so $5 \mid (7^2 - 2^2)$

   Induction Assume: $5 \mid (7^k - 2^k)$

   Show: $5 \mid (7^{k+1} - 2^{k+1})$

   Proof:
   $7^{k+1} - 2^{k+1} = 7 \cdot 7^k - 2 \cdot 2^k$
   
   $= 5 \cdot 7^k + 2 \cdot 7^k - 2 \cdot 2^k$
   
   $= 5 \cdot 7^k + 2(7^k - 2^k)$
   
   $= 5(7^k + 2m)$