Discrete Test 2 Review: first study quizzes!

(1) Let \( a, b \in \mathbb{Z} \). Prove that if \( a \mod 6 = 5 \) and \( b \mod 4 = 3 \) then \( 4a + 6b \mod 8 = 6 \).

(2) Suppose we were to prove or find a counterexample to the statement “\( \forall x \in S, y \in \mathbb{Z}, y \leq 25 \Rightarrow 5|(x + y) \).”

   (Answer without using the word “not” or the symbol “\( \sim \).”)

   a) For a direct proof we assume \( \quad \) and show \( \quad \).

   b) For proof using the contrapositive we assume \( \quad \) and show \( \quad \).

   c) For proof by contradiction we assume \( \quad \) and show that we reach a false conclusion.

   d) To disprove, using a counterexample, we find:

(3) Let \( a_1 = 2, a_2 = 4 \), and \( a_n = 5a_{n-1} - 6a_{n-2}, n \geq 3 \). Prove that \( \forall n \in \mathbb{N}, n \geq 3 \Rightarrow a_n = 2^n \) for all natural numbers \( n \).

(4) Use contradiction to prove: \( \forall a, b \in \mathbb{Z} \), if \( a \) is even and \( b \) is odd then 4 does not divide \( (a^2 + 2b^2) \).

   a) Negate the statement.

   b) Assuming that negation, prove that 4 divides 2.

(5) Prove that \( \sqrt{5} \) is irrational. You may assume that if \( 5|x^2 \) then \( 5|x \), by the F.T. of arithmetic.

(6) Prove \( \forall n \in \mathbb{Z}, n \geq 2 \Rightarrow \sum_{i=1}^{n} i^3 = \left( \frac{n(n+1)}{2} \right)^2 \).

(7) Disprove \( \forall n \in \mathbb{N}, (n^2 - n + 5) \) is prime.

(8) Prove that: \( \forall n \in \mathbb{N}, n \geq 2 \) then \( 3|(2^{4n-4}) + 2^{2n-3}) \).
(9) Given the one-time-pad sequence (2, 6, 13, 1) encrypt the word COOL. Your output will be letters.

(10) Use the BBS sequence to encrypt the word ZAP. Use the seed $a_0 = 11$ and the constant $pq = 7 \times 13 = 91$.

(11) Use the same BBS sequence to decrypt the word LLJ. Use the seed $a_0 = 11$ and the constant $pq = 7 \times 13 = 91$.

(12) Use the same BBS sequence to encrypt the digits 1101.

(13) Use the same BBS sequence to decrypt the digits 1110.