Ch. 9 cont.

Ex: How many whole numbers are there with exactly 3 digits?

* Same question as: How many PIN numbers are there with 3 digits, if the first digit cannot be zero?

Answer:

Three decisions: fill in these blanks

- - -

\[ \begin{array}{c}
\text{can be} \\
1-9 \\
\text{can be} \\
0-9
\end{array} \]

\[ \text{examples: } 752, 377, 401 \]

\[ X 047 \]

\[ \Rightarrow \text{Total} = 9 \cdot 10 \cdot 10 = 900 \]

Another way: Total number of numbers with up to 3 digits minus number of numbers with 2 or less:

\[ = 1000 - 100 = 900 \]

* These both count 0 as a whole number.

Ex: How many PIN numbers are there with 4 digits, this time the first can be zero, but no repeated digits are allowed?

Ans:

\[ \begin{array}{c}
\text{options} \\
10 \\
\text{options} \\
9 \\
\text{options} \\
8 \\
\text{options} \\
7
\end{array} \]

\[ = 5040 \]

\[ \text{examples: } 0521, 5073 \]

\[ X 7717 \]
Outline:
- Multiplication principle
  - \[ |A \times B| \]
  - Permutations

- Addition and subtraction principle
  - \[ |A \cup B| \]
  - \[ |U| - |B| \]
  - \[ |A \cup B \cup C \cup D| \]

- Division principle
  - \[ \frac{|U|}{|B|} \]

- Combinations
- Cyclic orders
- Binomial theorem

Permutation: How many 5 digit PIN's with no repeated numbers? (0 is ok)

\[
\begin{array}{cccccc}
10 & 9 & 8 & 7 & 6 \\
\end{array}
\]

\[= 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = 30,240\]

This kind of problem shows up a lot, so we have short hand:

\[
10^5 \overset{\text{5 decreasing numbers, ending at 10 - 5 + 1}}{=} 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = 30,240
\]

Ex: \[17^8 = 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 = 980,179,200\]