Calculus II. Review for Test 3 with answers.

1. For each series, what does the ratio test tell us? [converge absolutely, diverge, or inconclusive]. Show your work by finding the limit.
   (a) \( \sum_{n=1}^{\infty} \frac{(-2)^n}{n!} \)
   A: Limit = 0 < 1, so converges absolutely.
   (b) \( \sum_{n=1}^{\infty} \frac{(2^{n^2})}{(2n)!} \)
   A: Limit = \( \infty \) > 1, so diverges.

2. For each series, what does the root test tell us? [converge absolutely, diverge, or inconclusive]. Show your work by finding the limit.
   (a) \( \sum_{n=1}^{\infty} \left( \frac{-2n}{3n+1} \right)^n \)
   A: Limit = \( \frac{2}{3} \) < 1, so converges absolutely.
   (b) \( \sum_{n=1}^{\infty} \left( \frac{1}{\sqrt{n}} \right)^n \)
   A: Limit = 0 < 1, so converges absolutely.

3. For this power series, determine the radius of convergence.
   (a) \( \sum_{n=1}^{\infty} \frac{2(x - 3)^n}{n5^n} \)
   A: \( R = 5 \)

4. For this power series centered at \( a = 2 \), the radius of convergence is found to be \( R = 3 \). Determine the interval of convergence. (All you have to do is check the endpoints!)
   (a) \( \sum_{n=1}^{\infty} \frac{(x - 2)^n}{n3^{(n+1)}} \)
   A: \([-1, 5)\)
5. Find a power series for the following functions, by starting with a fact from the list of known Maclaurin series. Simplify just enough to combine the powers of $x$ into a single expression.

(a) $f(x) = e^{-x}$

A: $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$

(b) $f(x) = x^2 \sin(2x)$

A: $\sum_{n=0}^{\infty} \frac{(-1)^n 2(2n+1)x^{2n+3}}{(2n+1)!}$

(c) $f(x) = \frac{2x}{(1-x)^2}$

A: $\sum_{n=0}^{\infty} 2nx^n$

(d) $f(x) = \ln(1 + 2x)$

A: $\sum_{n=0}^{\infty} \frac{(-1)^n 2(n+1)x^{n+1}}{n+1}$

(e) $f(x) = \frac{3}{2-x}$

A: $\sum_{n=0}^{\infty} \frac{3x^n}{2(n+1)}$

6. Given $C = \left\{ \begin{array}{l} x = t^2 + 1 \\ y = e^{(t+2)} - t \end{array} \right\}$ $t \in [-3, 7]$.

Also given is that $y' = \frac{e^{(t+2)} - 1}{2t}$.

(a) Find the $(x, y)$ point(s) with horizontal tangent to the curve and use the second derivative to tell whether they are mins, maxes or inconclusive.

A: $(x, y) = (5, 3)$. At this point $y'' = \frac{1}{16} > 0$, so this point is a min.

(b) Set up the integral for the arc length of the curve.

$$A : \int_{-3}^{7} \sqrt{4t^2 + (e^{(t+2)} - 1)^2} \, dt$$
7. Given \( C = \begin{cases} 
  x = 1 - 2 \cos t \\
  y = -2 - \sin t 
\end{cases} \) for \( t \in [\pi/2, 3\pi/2] \).

(a) Find the Cartesian \( xy \)-equation that the curve obeys: eliminate \( t \).

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A : \frac{(x - 1)^2}{4} + (y + 2)^2 = 1.
\]

(b) Sketch \( C \). Use arrows and label the points for the first and last \( t \)-values.

(answer shown in class 4/20.)

8. For the polar point \((r, \theta) = (-3, 7\pi/6)\), find the \((x, y)\) coordinates. Graph the point.

A: \((x, y) = (3\sqrt{2}/2, 3\pi/4)\) (Graph shown in class.)

9. For the point \((x, y) = (-5\sqrt{2}, 5\sqrt{2})\) find the polar coordinates with positive radius \( r \).

A: \((r, \theta) = (10, 3\pi/4)\).

10. Graph the polar plot of \( r = 1 - \sin \theta \).

A: (answer shown in class 4/17.)

11. Find the \( n = 3 \) term of the Taylor series for \( f(x) = x^3 + e^{2x} \) centered at \( a = 5 \).

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A : \frac{(6 + 8e^{10})(x - 5)^3}{3!}
\]

12. Also study the quizzes, and the homework questions. These are good test questions too!