Calculus III. Review for Test 3, Spring ’15.

Also study all the homework and quizzes, as well as examples in class notes.

NOTE: Some questions on the actual test may state “Set up the integral only.” Since you don’t know which kind, for practice do both the set-up and the integration.

1. Find the integral\[
\int_1^2 \int_{-\pi}^{\sqrt{4\pi-x^2}} \int_0^{\sqrt{\pi}} (2 + 3 \sin(x^2 + y^2)) \, dy \, dx \, dz.
\]

First show the set up of the integral, using only polar variables.

For the following two problems let \( \vec{F}(x, y, z) = \langle 2z, x^2y, y + x \ln z \rangle \).

2. \[\n\text{Find } \nabla \cdot \vec{F} \text{ at } (1, 0, 2). \text{ Source or sink?} \]
   The function \( \nabla \cdot \vec{F}(x, y, z) = \) __________

3. \[\n\text{Find } \nabla \times \vec{F} \text{ at } (1, 0, 1). \]
   The field \( \nabla \times \vec{F}(x, y, z) = \) __________

4. Find \( \int_C \nabla f \cdot d\vec{r} \), where \( f(x, y, z) = z + \sin(\ln(xy)) \) and \( C \) be the line from \( (\frac{1}{3}, 3, 7) \) to \( (2, \frac{1}{2}, 11) \).

5. Find the volume inside \( \rho = 3 \) from \( \phi = 0 \) to \( \frac{\pi}{2} \) and \( \theta = 0 \) to \( \frac{\pi}{4} \).

6. Let \( \vec{F} = \langle 1, ze^y, e^y \rangle \) and \( C \) be the curve given by \( \vec{r}(t) = \langle \sin(\frac{\pi t}{2}), t^3 + t^2 + t, e^{3t} \rangle; 0 \leq t \leq 1. \)
   (a) Find the curl of \( \vec{F} \).
   (b) Find \( f \) such that \( \vec{F} = \nabla f \).
   (c) Find \( \int_C \vec{F} \cdot d\vec{r} \).
7. Let \( \vec{F} = \langle y^2 + 5 + \sin^2(e^x), 2x + \sin^2 y, 0 \rangle \) and \( C \) be the triangular path in the \( xy \)-plane from \((0,0)\) to \((1,0)\) to \((1,2)\) and back to \((0,0)\).
(a) Find the curl of \( \vec{F} \).
(b) Find \( \int_C \vec{F} \cdot d\vec{r} \).

8. Let \( \vec{F} = \langle e^{3x} - 3y, 5y^3 + 1, 0 \rangle \) and \( C \) be the counter-clockwise circle \( x^2 + y^2 = 9 \).
(a) Find the curl of \( \vec{F} \).
(b) Find \( \int_C \vec{F} \cdot d\vec{r} \).

9. Let \( \vec{F} = \langle e^z, 0, x + y \rangle \) and \( C \) be the line segment from \((1,2,1)\) to \((3,1,2)\).
(a) Find the curl of \( \vec{F} \).
(b) Find \( \int_C \vec{F} \cdot d\vec{r} \).

10. Integrate the function \( f(x,y,z) = 2x \) over the tetrahedron with vertices \((0,0,0)\), \((0,1,0)\), \((0,1,3)\) and \((4,1,0)\).