Calculus III. Test 1 Review.

Also make sure to study your 3 quizzes!

1. Given \( P = (5, 3, -1); \quad Q = (-2, 1, 0); \quad \vec{r}(t) = \langle \sin(e^{3t} - 1), 7(2^t), \ln(e^t + 5) \rangle. \)

(a) Find the parametric equations for the line that goes through \( Q \) and is perpendicular to the plane \( 7z - 4y = 11x + 21. \)

(b) Find the plane through point \( P \) and perpendicular to the line \( -x = 4y = \frac{z}{2}. \) Simplify by collecting the constants on the right of your answer.

(c) Find the parametric equations for the tangent line to \( \vec{r}(t) \) at \( t = 0. \)
2. Given for a spaceship:
\[ \mathbf{r}(3) = \langle 0, -11, -8 \rangle, \quad \mathbf{r}'(3) = \langle 1, 2, 1 \rangle, \quad \mathbf{T}'(3) = \langle 1, 0, -1 \rangle, \quad a_T(3) = 5. \]

(a) Find \( \kappa(3) \).
(b) Find \( \mathbf{T}(3) \).
(c) Find \( a_N(3) \).
(d) Find \( \mathbf{N}(3) \).
(e) Find the speed at \( t = 3 \).
(f) Is the spaceship speeding up or slowing down at \( t = 3 \)?
(g) Find the acceleration at \( t = 3 \).
3. Given $\vec{r}(t) = (t^2 + t, 5, -3t)$.
   
   (a) Find the $t$-value of the max curvature.

   (b) Find $a_T(1)$.

   (c) Find $a_N(2)$.

   (d) Find the velocity at $t = 3$.

   (e) Set up the integral for the arc length from $t = 0$ to $t = 5$.

   (f) Is the spaceship speeding up or slowing down at $t = 1$?
4. Given for a spaceship located at $\vec{r}(3) = \langle 9, 0, -4 \rangle$:
   
   $\vec{a}(3) = \langle 0, -11, -8 \rangle$, $\vec{T}(3) = \langle 0, 1, 0 \rangle$, $\vec{N}(3) = \langle 0, 0, -1 \rangle$, and speed $= \frac{1}{4}$.

   (a) Find $a_T(3)$.  (d) Is the spaceship speeding up or slowing down at $t = 3$?
   (b) Find $\kappa(3)$.  (e) Find $\vec{v}(3)$.
   (c) Find $a_N(3)$.  (f) Find parametric equations for the tangent line at $t = 3$. 
5. Given:
\[ \vec{r}(3) = \langle 11, -8, 0 \rangle, \quad \vec{r}'(3) = \langle 3, 0, -2 \rangle, \quad \vec{N}(3) = \langle 0, 1, 0 \rangle, \quad a_N(3) = 2, \quad \text{and} \quad a_T(3) = -4. \]

(a) Find the tangent line to the curve \( \vec{r}(t) \) at \( t = 3 \). Give parametric equations for the line.

(b) Find the acceleration \( \vec{a}(3) \).

(c) Is the spaceship speeding up or slowing down at \( t = 3 \)?