1) Given the preposet $P$, where $a \rightarrow b$ means $a \leq b$, find the list $\mathcal{F}$ of open sets in $P_{\text{top}}$.

2) Given the topological space $X$, where a circled subset means it is an open set, find the preposet $p(X)$. Draw it as a digraph, with $a \rightarrow b$ meaning $a \leq b$, (and you can assume reflexivity).

3) Is $X$ a $T_0$ space?
II.

1) Let $X = \{1, 2, 3, 4\}$
and $\mathcal{F} = \{\emptyset, \{1, 3\}, \{3, 4\}, \{1, 2, 3, 4\}\}$

a) Explain why $X, \mathcal{F}$ does not make a topological space.

b) If we tried to find $\mathcal{P}(X)$, what rule of preorders would be broken?

2) Given two Hasse diagrams:

$P$:

```
          a
          |
         --
          b
          |
         --
       c
       |
     --
     e
     |
   --
g
```

$Q$:

```
        a
        |
      --
      b
      |
    --
    c
    |
  --
d
  |
- e
```

→ Find $Q_{top}$.
→ Find a max-length chain and antichain for both $P$ and $Q$.
→ Is $P \preceq Q$ as posets?
→ Which one is a lattice? Find $c \wedge e$.
→ Let $f: P \rightarrow Q$ be given by:

\[ f(a) = f(c) = f(b) = a \]
\[ f(d) = f(e) = f(g) = g \]

Is $f: P_{top} \rightarrow Q_{top}$ continuous?
III. 
1) Draw the Boolean lattice using the set \( \{1, 2, 3\} \). We'll call it \( B_3 \).

2) In the topology \((B_3)_{\text{top}}\), what are the nbhds of \( \{1, 3\} \)?

3) Recall that the maximal tree in the Tamari lattice \( Y_4 \) looks like:

\[
\begin{array}{c}
\text{In the topology } (Y_4)_{\text{top}}, \text{ what are the nbhds of } Y_4? \\
\end{array}
\]

4) Find the length of the longest chain in \( B_n \).

5) Recall that \( |B_n| = 2^n \) and \( |Y_n| = \frac{1}{n} \binom{2n-2}{n-1} \).

How many functions are there from \( B_n \) to \( Y_n \)?