Individual Reading: 3450:697 The Associahedron: topology, combinatorics, and applications. Instructor: Stefan Forcey

This is a reading course. The students will turn in two kinds of assignment: typed notes on basic math, and typed responses to 9 journal articles. The notes will follow an outline, as given below. The responses will be to articles or sections of articles, and will consist of a half page or less summary and a half page or less of questions. The summary will be no more than a first impression. The questions may include "What is a …?" about terminology, and at least one question about the implications of the article: "What if the definition were changed…would the theorem still hold?" Only one of the optional project items needs be chosen, (this may make an optional article and some definitions necessary.)

Instructions: Use Wikipedia and Mathworld for basic definitions, articles for specific definitions. Turn in notes by deadlines. Turn in one page article response each week, starting week 2. Choose one project from either part I:G-J II:C-D or from parts III-V. This project may be turned in as a poster for presentation, or as a short (two page) paper.

Grading: There are 6 non-optional outline items worth 1 point each, a project worth 2 points and 9 articles worth 1 point each. Grading will be based on completion: the grade assigned will be the fraction (percentage) of the 18 total points completed satisfactorily. A=90%, B=80%, C=70% D=60% F=50% Also + ,- will be up to the professor.

The reading:

1) Here are some shapes!
http://www.ams.org/featurecolumn/archive/associahedra.html

2) and more…
http://www.claymath.org/library/academy/LectureNotes05/Lodaypaper.pdf

3) a general introduction to associahedra:
http://www.math.uakron.edu/~sf34/pattern_kent.pdf

4) **A REALIZATION OF GRAPH-ASSOCIAHEDRA**
http://web.williams.edu/go/math/devadoss/files/realization.pdf

5) Section 7 of **PSEUDOGRAPH ASSOCIAHEDRA**
http://www.math.uakron.edu/~sf34/pseudograph.pdf

6) Algebraic Topology, A. Hatcher, Chapter 0
http://www.math.cornell.edu/~hatcher/AT/ATch0.pdf
source: http://www.math.cornell.edu/~hatcher/AT/ATpage.html (CW-complexes, page 5.)

Optional: J. D. Stasheff,H-spaces from a homotopy point of view, Lecture Notes in Mathematics 161, Springer-Verlag, Berlin (1970), ii-95

Optional: Convex Hull Realizations of the Multiplihedra
Topology and its Applications, 156, 326-347, 2008. [arXiv] [journal]

Optional: Quotients of the multiplihedron as categorified associahedra.
Homotopy, Homology and Applications, vol. 10(2), 227–256, 2008. [arXiv] [journal]

Optional: Marked tubes and the graph multiplihedron. (with S.L. Devadoss)
Algebraic and Geometric Topology , 8(4) 2081–2108, 2008. [arXiv] [journal]

7) The Mathematics of Phylogenomics, Lior Pachter, Bernd Sturmfels, Section 7 Phylogenetic Combinatorics.
http://arxiv.org/PS_cache/math/pdf/0409/0409132v2.pdf

8) The Neighbor-Net Algorithm, Dan Levy, Lior Pachter Section 2 The Mathematics.
http://arxiv.org/PS_cache/math/pdf/0702/0702515v2.pdf

8.5) Topology of Reticulate Evolution Kevin Joseph Emmett
http://academiccommons.columbia.edu/download/fedora_content/download/ac:198974/CONTENT/Emmett_columbia_0054D_13361.pdf

9) KP SOLITONS AND TOTAL POSITIVITY FOR THE GRASSMANNIAN Kodamaa and Williams, Section 10.
http://arxiv.org/PS_cache/arxiv/pdf/1106/1106.0023v1.pdf

Optional: Kodamaa and Williams KP solitons, total positivity, and cluster algebras,PNAS 2011
http://www.pnas.org/content/early/2011/05/10/1102627108.full.pdf+html

Plan of Notes: These may be typed in Latex or Word. Sources must be cited for definitions and examples. Definitions must be self-contained, examples must be self-contained as well as elementary as possible!

I. Combinatorics. Additional source (Loday): http://arxiv.org/PS_cache/math/pdf/0212/0212126v1.pdf
Finish by: ____ Sep 23_____.

A. Define: K(n), the nth associahedron, using binary trees and convex hulls. How are the associahedra related to each other?
B. Define the permutohedron and the simplex.
C. Optional: Define: J(n), the nth multiplihedron. How are the multiplihedra related to each other and the associahedra?
D. Optional: Define CK(n), the nth composihedron. How are the composihedra related to each other and the associahedra?
E. Define the cylohedron, graph-associahedron, and pseudograph-associahedron. Relate to the associahedron, permutohedron and simplex.
F. Expand the proof that truncation of a simplex can give the pseudograph associahedra for all graphs.
G. Optional Project: define a multi-building set and show that the pseudograph associahedra give examples.
H. Optional Project: Construct five different graph and pseudograph associahedra. Construct 5 different graph and pseudograph multiplihedra. Conjecture on the relationships.
I. Optional Project: measure the lengths of edges and the total volumes of various faces of the convex hull realizations of the multiplihedra. Also compute centroids. Conjecture on the formula for finding these in general. Repeat for the composihedra.
J. Optional Project: develop generating functions and combinatorial formulas for the face vectors of the multiplihedra, composihedra, etc.

II. Topology. http://en.wikipedia.org/wiki/Homotopy ; http://en.wikipedia.org/wiki/Path_(topology)
Finish by:____Oct 28_____.

A. Define: topological space, continuous map, path.
B. Define: topological sphere, CW-complex, homotopy of maps, homotopy of paths.
C. Optional Project: Define: H-space and A_n space. Relate to associahedron.
D. Optional Project: Define: A_n space morphism. Relate to multiplihedron.

III. Optional Project: Application to phylogenetics.
Finish by :____Nov 28_____.

A. Define a metric space and a tree-metric.
B. Define the neighbor-joining algorithm.
C. Define the neighbor-net algorithm.
D. Construct phylogenetic trees and networks for two sample metrics.

IV. Optional Project: Application to soliton waves. Additional source: http://arxiv.org/PS_cache/arxiv/pdf/1110/1110.3507v1.pdf

A. Define the soliton (KP) wave equation.
B. Describe the map from associahedra faces to soliton graphs.
C. Construct wave contour plots corresponding to five triangulated polygons.

V. Optional Project: Application to Quantum Electrodynamics

A. Define the noncommutative Connes-Kreimer Hopf algebra of Feynman diagrams.
B. Prove that the Connes-Kreimer algebra is isomorphic to the Loday-Ronco algebra.
C. Find the antipode of four different Feynman diagrams.

VI. Optional Project: Higher categorical structures and Hopf algebras.

A. Category Theory review
B. Bicategories and tricategories.
C. Species, Posets, Operads and PROPS
D. Incidence algebras
E. Operad algebras
F. Project: Define a procedure for turning an operad module into a Hopf module.