

$$\left. \begin{array}{l} m^2 - 8m + 16 \\ (m-4)^2 \end{array} \right\} \begin{array}{l} e^{4t} \\ t e^{4t} \end{array}$$

✓

$$x^2 y'' - 7xy' + 16y = 0$$

$$y_1 = x^4$$

$$y_1' = 4x^3$$

4 pt

$$\left\{ \begin{array}{l} y = u y_1 \\ y' = u' y_1 + u y_1' \\ y'' = u'' y_1 + 2u' y_1' + u y_1'' \end{array} \right.$$

6 pt

$$x^2 (u'' y_1 + 2u' y_1' + \cancel{u y_1''}) - 7x (u' y_1 + \cancel{u y_1'}) + 16 \cancel{u y_1} = 0$$

$$\frac{x^2 y_1}{x^6} u'' + \underbrace{(2x^2 y_1' - 7x y_1)}_{x^5} u' = 0$$

$$w = u'$$

$$x^6 w' + x^5 w = 0$$

$$x w' + w = 0$$

$$\frac{d}{dx} \{xw\} = 0$$

$$xw = c_1$$

$$u' = w = \frac{c_1}{x} \Rightarrow u = c_1 \ln x + c_2$$

5 pt

3 pt

$$y = (c_1 \ln x + c_2) x^4 = \begin{array}{l} c_1 x^4 \ln x + c_2 x^4 \\ \vdots \end{array}$$

$$y = x^4 \ln x$$

Quiz 5B Key

$$m^2 - 3m - 4 = (m-4)(m+1)$$

ivSiiQe $e^{-t} \quad e^{4t}$

$$x^2 y'' - 2xy' - 4y = 0$$

$$y_1 = \frac{1}{x}$$

$$y_1' = -\frac{1}{x^2}$$

2 pt

$$y = u y_1$$

$$y' = u' y_1 + u y_1'$$

$$y'' = u'' y_1 + 2u' y_1' + u y_1''$$

$$x^2(u'' y_1 + 2u' y_1' + \cancel{u y_1''}) - 2x(u' y_1 + \cancel{u y_1'}) - 4 \cancel{u y_1} = 0$$

3 pt

$$\underbrace{x^2 y_1}_{x} u'' + \underbrace{(2x^2 y_1' - 2x y_1)}_{-2} u' = 0$$

$$w = u'$$

$$x w' - 4w = 0$$

$$w' - \frac{4}{x} w = 0$$

$$e^{\int (-\frac{4}{x}) dx} = e^{-4 \ln|x|} = x^{-4}$$

$$x^{-4} w' - 4x^{-5} w = 0$$

$$\frac{d}{dx} (x^{-4} w) = 0$$

$$x^{-4} w = c_1 \Rightarrow w = c_1 x^4 \Rightarrow u = \frac{c_1}{5} x^5 + c_2$$

2 pt

$$y = \left(\frac{c_1}{5} x^5 + c_2 \right) \frac{1}{x} = \frac{c_1}{5} x^4 + \frac{c_2}{x} = x^4$$