Consider $c_1 x^3 + c_2(1+x) + c_3 \cdot 7 = 0$ #

or $c_1 x^3 + c_2 x + c_2 + 7c_3 = 0$

Solution 1:

$\boxed{x=0}$ $c_2 + 7c_3 = 0$ (1)

$\boxed{x=1}$ $c_1 + c_2 + c_2 + 7c_3 = 0$ (2)

$\boxed{x=2}$ $8c_1 + 2c_2 + c_2 + 7c_3 = 0$ (3)

(1) & (2) $\Rightarrow$ $c_1 + c_2 = 0$ (4)

(1) & (3) $\Rightarrow$ $8c_1 + 2c_2 = 0$ (5)

(4) $\Rightarrow$ $c_1 = -c_2$

So (5) becomes $-8c_2 + 2c_2 = 0 \Rightarrow c_2 = 0$

Then $c_1 = 0$ & (1) $\Rightarrow$ $c_3 = 0$.

So # $\Rightarrow$ $c_1 = c_2 = c_3 = 0 \Rightarrow f_1, f_2, f_3$ li

Solution 2: $c_1 x^3 + c_2 x + c_2 + 7c_3 = 0$ for all x

implies $c_1 = 0$, $c_2 = 0$, $c_2 + 7c_3 = 0$

then $0 + 7c_3 = 0 \Rightarrow c_3 = 0$.

$c_1 = c_2 = c_3 = 0 \Rightarrow f_1, f_2, f_3$ li

Consider $c_1(-x) + c_2 x^2 + c_3(1+x) = 0$

Regroup: $c_2 x^2 + (c_3 - c_1)x + c_3 = 0$

Solution 1:

$\boxed{x=0} \Rightarrow c_3 = 0$

$\boxed{x=1} \Rightarrow c_2 - c_1 = 0$

$\boxed{x=-1} \Rightarrow c_2 + c_1 = 0$

Add these $\quad 2c_2 = 0 \Rightarrow c_2 = 0$

Then $c_1 = 0$. So $c_1 = c_2 = c_3 = 0$.

So f_1, f_2, f_3 are li.

Solution 2:

$c_2 x^2 + (c_3 - c_1)x + c_3 = 0$

$\Rightarrow c_2 = 0, \quad c_3 - c_1 = 0, \quad c_3 = 0$

Since $c_3 = 0$, this eq. $\Rightarrow c_1 = 0$.

So $c_1 = c_2 = c_3 = 0$. So f_1, f_2, f_3 are li.