iiSivQj ·

1.

$$\frac{\partial}{\partial y}\left\{x^2y^3 - \frac{1}{1+x^2}\right\} = 3x^2y^2$$

4pts

$$= \checkmark \quad \text{So eqn. is exact.}$$

$$\frac{\partial}{\partial x}\left\{x^3y^2\right\} = 3x^2y^2$$

Now, find f s.t.

$$\frac{\partial f}{\partial x} = x^2y^3 - \frac{1}{1+x^2} \quad \& \quad \frac{\partial}{\partial y}f = x^3y^2$$

4pts So $f = \frac{1}{3}x^3y^3 - \tan^{-1}x + g(y)$

So

4pts $x^3y^2 + g' = \frac{\partial f}{\partial y} = x^3y^2 \Rightarrow g' = 0 \Rightarrow g = \text{const.}$ 2pts

So $f = \frac{1}{3}x^3y^3 - \tan^{-1}x = C$

and impl. sol. is $\frac{1}{3}x^3y^3 - \tan^{-1}x = c$ 2pts iiSiiQw

2. $$\frac{dy}{dx} = \frac{y^{1/2} x e^{-x}}{y+1}$$

$$\int \frac{y+1}{y^{1/2}}\, dy = \int x e^{-x}\, dx$$ 8 pt

4 pt

$$\int \left[y^{1/2} + y^{-1/2} \right] dy = -x e^{-x} - e^{-x} + c$$

4 pt $$\frac{2}{3} y^{3/2} + 2 y^{1/2} = -x e^{-x} - e^{-x} + c$$

3. iiSiiiQd

$$x^2 y' + x(x+2)y = e^x \qquad y(-1) = 3$$

st. form

2pt $$y' + \frac{x+2}{x} y = \frac{1}{x^2} e^x$$

Construct IF 4pt

$$e^{\int \frac{x+2}{x}} = e^{\int 1 + \frac{2}{x}}$$

$$= e^{x + 2\ln|x|}$$

$$= e^x e^{\ln|x|^2}$$

$$= e^x |x|^2 = x^2 e^x$$

Mult. thru by IF

6pt

$$x^2 e^x y' + x(x+2) e^x y = e^{2x}$$

$$\Rightarrow \int \frac{d}{dx}\{x^2 e^x y\} dx = \int e^{2x}\, dx$$

$$\Rightarrow x^2 e^x y = \frac{1}{2} e^{2x} + c$$

$$\Rightarrow y = \frac{e^x}{2x^2} + \frac{c}{x^2 e^x}$$

2pt.

Find c.

$$3 = y(-1) = \frac{e^{-1}}{2} + \frac{c}{e^{-1}} = \frac{1}{2e} + ce$$

$$\Rightarrow c = \frac{1}{e}\left(3 - \frac{1}{2e}\right)$$

So,

$$y = \frac{e^x}{2x^2} + \frac{1}{e}\left(3 - \frac{1}{2e}\right)\frac{1}{x^2 e^x}$$

2pts $I_0 = (-\infty, 0)$

iiSvQi

4.

$$y^{1/2}\frac{dy}{dx}+y^{3/2}=1 \qquad y(0)=4$$

$$\frac{dy}{dx}+y=y^{-\frac{1}{2}}$$

$$u=y^{1-(-\frac{1}{2})}=y^{3/2}$$ 2 pt

$$y=u^{2/3}$$ 2 pt $$\frac{dy}{dx}=\frac{2}{3}u^{-\frac{1}{3}}\frac{du}{dx}$$ 2pt

$$\frac{2}{3}u^{-\frac{1}{3}}\frac{du}{dx}+u^{2/3}=u^{-\frac{1}{3}}$$

2 pt $$\frac{du}{dx}+\frac{3}{2}u=\frac{3}{2}$$ $$I.F.=e^{\int\frac{3}{2}dx}=e^{\frac{3}{2}x}$$

$$e^{3/2x}\frac{du}{dx}+\frac{3}{2}e^{3/2x}u=\frac{3}{2}e^{\frac{3}{2}x}$$

$$\int\frac{d}{dx}\left\{e^{\frac{3}{2}x}u\right\}dx=\int\frac{3}{2}e^{\frac{3}{2}x}dx$$

$$e^{\frac{3}{2}x}u=e^{\frac{3}{2}x}+c$$

$$u=1+ce^{-\frac{3}{2}x}$$

4 pts

$$y=u^{2/3}=\left(1+ce^{-\frac{3}{2}x}\right)^{2/3}$$

$$4=y(0)=(1+c)^{2/3} \Rightarrow c=7$$

$$y=\left(1+7e^{-\frac{3}{2}x}\right)^{2/3}$$ 2 pts

5. iiSvQe

$$\frac{dy}{dx} = \frac{y-x}{y+x}$$

or $(x-y)\,dx + (y+x)\,dy = 0$

$y = ux \qquad dy = x\,du + u\,dx$ 2 pt

$(x-ux)\,dx + (ux+x)(x\,du + u\,dx) = 0$

$(x - \cancel{ux} + u^2x + \cancel{ux})\,dx + (ux+x)x\,du = 0$ 6 pt

$x(1+u^2)\,dx + x^2(u+1)\,du = 0$

$$\int\left(\frac{u+1}{1+u^2}\right)du = \int -\frac{1}{x}\,dx$$

$$= \int\left(\frac{u}{1+u^2} + \frac{1}{1+u^2}\right)du$$

4 pt

$\frac{1}{2}\ln(1+u^2) + \tan^{-1}u = -\ln|x| + C$ 2 pt

$\frac{1}{2}\ln\left(1+\frac{y^2}{x^2}\right) + \tan^{-1}\left(\frac{y}{x}\right) = -\ln|x| + C$ 2 pt

6.

(a) $\dfrac{dT}{dt} = k(T-75)$ 4 pt

(b) $\dfrac{dT}{dt} - kT = -75k$ IF e^{-kt}

$$e^{-kt}\frac{dT}{dt} - e^{-kt}kT = -75ke^{-kt}$$

$$\int \frac{d}{dt}\left\{e^{-kt}T\right\}dt = \int (-75ke^{-kt})\,dt$$

$$e^{-kt}T = 75e^{-kt} + c$$

$$T = 75 + ce^{kt}$$ 2 pt

Find c: $120 = T(0) = 75 + c \Rightarrow c = 45$

So $T = 75 + 45e^{kt}$

Find k: $105 = T(10) = 75 + 45e^{k\cdot 10}$

or $30 = 45e^{10k}$

or $k = \frac{1}{10}\ln\left(\frac{2}{3}\right)$

So $T = 75 + 45e^{\frac{1}{10}\ln\left(\frac{2}{3}\right)t}$

$= 75 + 45\left(\frac{2}{3}\right)^{t/10}$ 2 pt

(c) $T(40) = 75 + 45\left(\frac{2}{3}\right)^{40/10}$ 2 pt iiSvQz

7.

$$x^2 y \frac{dy}{dx} + x y^2 = x^2 y^2$$

$$x^2 y \, dy + (x y^2 - x^2 y^2) dx = 0$$

$$M_y \neq N_x$$

$$\frac{dy}{dx} + \frac{1}{x} y = y$$

$$\frac{dy}{dx} + \left(\frac{1}{x} - 1\right) y = 0$$

$$\frac{1}{y} dy = \left(1 - \frac{1}{x}\right) dx$$

lin.	☑	2 pt
sep.	☑	2 pt
exact	☐	2 pt
homog.	☐	2 pt
Bern.	☑	2 pt