Series Examples

1. \(\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}\) converges conditionally, by the Alternating Series Test, since the series is an alternating \(p\)-series with \(p = 1\).

2. \(\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{2n-1}}\) converges conditionally by the Alternating Series, and Limit Comparison Tests.

3. \(\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{e^n}\) converges absolutely, by the Ratio Test.

4. \(\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}\) converges conditionally, by the Comparison and Alternating Series Tests.

5. \(\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3n}\) diverges, by the Test for Divergence.

6. \(\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2 + 1}\) converges conditionally by the Limit Comparison and Alternating Series Tests.

7. \(\sum_{n=2}^{\infty} \frac{(-1)^n \ln n}{n}\) converges conditionally, by the Comparison and Alternating Series Tests.

8. \(\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}\) converges absolutely, by the Ratio Test.

9. \(\sum_{n=1}^{\infty} (-1)^{n+1} 2^{1/n}\) diverges, by the Test for Divergence.

10. \(\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \sqrt{2n-1}}{n}\) converges conditionally, by the Limit Comparison and Alternating Series Tests.

11. \(\sum_{n=1}^{\infty} a_n, \text{ where } a_n = \begin{cases} \frac{-1}{n^2} & \text{if } n \text{ is a square} \\ \frac{1}{n^2} & \text{otherwise} \end{cases}\) converges, since it is the difference of two \(p\)-series.

12. \(\sum_{n=1}^{\infty} a_n, \text{ where } a_n = \begin{cases} \frac{-1}{n^2} & \text{if } n \text{ is a multiple of 3} \\ \frac{1}{n^2} & \text{otherwise} \end{cases}\) diverges, since it is the difference of a convergent \(p\)-series, and a multiple of the Harmonic series.

13. \(\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}\) converges conditionally by the Alternating Series Test, since it is an alternating \(p\)-series.

14. \(\sum_{n=1}^{\infty} (-1)^{n+1} \left(1 + \frac{1}{n}\right)^{-n}\) diverges, by the Test for Divergence.
15. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(e^n + e^{-n})}$ converges conditionally, by the Limit Comparison and Alternating Series Tests.

16. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln \left(1 + \frac{1}{n}\right)}$ diverges, by the Test for Divergence.

17. $\sum_{n=1}^{\infty} (-1)^n \int_{n}^{n+1} \frac{e^{-x}}{x} \, dx$ converges absolutely, by the Comparison Test.

18. $\sum_{n=1}^{\infty} (-1)^n n \sin \left(\frac{1}{n}\right)$ diverges, by the Test for Divergence.

19. $\sum_{n=1}^{\infty} \frac{\sin(n\pi/4)}{n!}$ converges absolutely, by the Comparison Test.

20. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2^n}{n!}$ converges absolutely, by the Ratio Test.

21. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{2n}$ converges absolutely, by the Ratio Test.

22. $\sum_{n=1}^{\infty} n! \left(\frac{1}{3}\right)^n$ diverges, by the Ratio Test.

23. $\sum_{n=1}^{\infty} \frac{n + 2}{n(3^n)}$ converges absolutely, by the Ratio Test.

24. $\sum_{n=1}^{\infty} \frac{n^2}{2^{n}}$ converges absolutely, by the Ratio Test.

25. $\sum_{n=1}^{\infty} \frac{n}{e^{nx}}$ converges absolutely, by the Root Test.

26. $\sum_{n=1}^{\infty} \frac{1}{e^n}$ converges absolutely, since it is a convergent geometric series.

27. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{2n + 1}}$ diverges, by the Limit Comparison Test.

28. $\sum_{n=1}^{\infty} \frac{1}{(2n + 1)^2}$ converges absolutely, by the Limit Comparison Test.

29. $\sum_{n=1}^{\infty} \frac{1}{8n}$ diverges, by the Limit Comparison Test.

30. $\sum_{n=1}^{\infty} \frac{5}{n^{17}}$ diverges, by the Limit Comparison Test.
31. $\sum_{n=1}^{\infty} \frac{100^n}{n!}$ converges absolutely, by the Ratio Test.

32. $\sum_{n=2}^{\infty} (\sqrt{n} - 1)^n$ converges absolutely, by the Root Test.

33. $\sum_{n=1}^{\infty} \frac{2^n + 3^n}{6^n}$ converges absolutely, since it is the sum of two convergent geometric series.

34. $\sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n}}{n^2 + 1}$ converges, by the Limit Comparison Test.

35. $\sum_{n=1}^{\infty} \frac{1}{n \ln n [\ln \ln n]}$ diverges, by the Integral Test.

36. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ converges absolutely, by the Limit Comparison Test.

37. $\sum_{n=1}^{\infty} \frac{n+1}{(n+2)(n+3)(n+4)}$ converges absolutely, by the Limit Comparison Test.

38. $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$ converges absolutely, by the Ratio Test.

39. $\sum_{n=1}^{\infty} \frac{1}{1 + \sqrt{2}}$ diverges, by the Test for Divergence.

40. $\sum_{n=1}^{\infty} \frac{1}{2n - 1}$ diverges, by the Limit Comparison Test.

41. $\sum_{n=1}^{\infty} \frac{1}{1 + n^2}$ converges absolutely, by the Limit Comparison Test.

42. $\sum_{n=2}^{\infty} \frac{\ln n}{n}$ diverges, by the Comparison Test.

43. $\sum_{n=2}^{\infty} \frac{1}{(\ln n)^{3n}}$ diverges, by the Test for Divergence.

44. $\sum_{n=2}^{\infty} \frac{2^{n-1}}{(n-1)!}$ converges absolutely, by the Ratio Test.

45. $\sum_{n=1}^{\infty} \frac{3^{3n}}{(3n)!}$ converges absolutely, by the Ratio Test.

46. $\sum_{n=1}^{\infty} \frac{3n}{n^2 + 1}$ diverges, by the Limit Comparison Test.
47. \( \sum_{n=0}^{\infty} \frac{1}{1 + \sqrt{n}} \) diverges, by the Limit Comparison Test.

48. \( \sum_{n=1}^{\infty} \frac{n}{n^2 + 1} \) diverges, by the Limit Comparison Test.

49. \( \sum_{n=1}^{\infty} \frac{n}{2^n} \) converges absolutely, by the Ratio Test.

50. \( \sum_{n=1}^{\infty} \frac{2n - 1}{n^2 + n} \) diverges, by the Limit Comparison Test.

51. \( \sum_{n=2}^{\infty} \frac{1}{\sqrt{n} - 1} \) diverges, by the Limit Comparison Test.

52. \( \sum_{n=1}^{\infty} \frac{n!}{n^2(n+1)^2} \) diverges, by the Ratio Test.

53. \( \sum_{n=1}^{\infty} \frac{2^n n!}{n^n} \) converges absolutely, by the Ratio Test (Tricky!).

54. \( \sum_{n=1}^{\infty} \frac{n}{n^3 + 4} \) converges absolutely, by the Limit Comparison Test.

55. \( \sum_{n=1}^{\infty} \frac{1}{n^2 \cos(\pi/n)} \) converges absolutely, by the Limit Comparison Test.

56. \( \sum_{n=1}^{\infty} \frac{n^2}{n^3 + 1} \) diverges, by the Limit Comparison.

57. \( \sum_{n=4}^{\infty} \frac{1}{n^2 - 9} \) converges absolutely, by the Limit Comparison Test.

58. \( \sum_{n=1}^{\infty} \frac{n}{n!} \) converges absolutely, by the Ratio Test.

59. \( \sum_{n=1}^{\infty} \frac{1}{n^2 + 1} \) converges absolutely, by the Limit Comparison Test.

60. \( \sum_{n=1}^{\infty} \frac{1}{4n + 3} \) diverges, by the Limit Comparison Test.

61. \( \sum_{n=6}^{\infty} \frac{1}{(n-3)(n-5)} \) converges absolutely, by the Limit Comparison Test.

62. \( \sum_{n=1}^{\infty} \frac{1}{\left(\frac{n + \frac{1}{n}}{n}\right)^n} \) converges absolutely, by the Root Test.
63. \( \sum_{n=2}^{\infty} \frac{1}{n \ln n} \) diverges, by the Integral Test.

64. \( \sum_{n=1}^{\infty} \frac{1}{n \sqrt{n}} \) is an absolutely convergent p-series.

65. \( \sum_{n=1}^{\infty} \frac{n}{e^n} \) converges absolutely, by the Ratio Test.

66. \( \sum_{n=2}^{\infty} \frac{\ln n}{n^2} \) converges absolutely, by the Integral Test.

67. \( \sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2 + 1}} \) diverges, by the Test for Divergence.

68. \( \sum_{n=1}^{\infty} \frac{n \sqrt{2}}{n^3 + 1} \) converges absolutely, by the Limit Comparison Test.

69. \( \sum_{n=2}^{\infty} \left( \frac{3n^3 + 4n^2 - 7}{\sqrt{4n^6 + 9n^4 - 10}} \right)^n \) diverges, by the Root Test.