1. (a) Write \( \frac{2 + i}{(1 - i)(1 + 2i)} \) in the form \( a + bi \).

(b) Use the polar/exponential form to evaluate \((-1 + \sqrt{3}i)^5\) in the form \( a + bi \).

(c) Use Euler’s formula and de Moivre’s Theorem to write \( \cos 3\theta \) in terms of \( \cos \theta \) and \( \sin \theta \).

(d) Sketch the region \( 1 \leq |z + 2 - i| < 2 \).

2. (a) Describe the difference between the statements ”\( f(z) \) is differentiable at \( z = z_0 \)” and ”\( f(z) \) is analytic at \( z = z_0 \)”.

(b) Use limits to determine if \( f(z) = |z| \) is differentiable at \( z = 0 \).

3. (a) Describe the Cauchy-Riemann equations.

(b) Let \( v = e^x \sin y + e^{-y} \sin x \). Find a function \( u \) such that \( f(z) = u(x, y) + i \, v(x, y) \) is entire.

4. Using the definition, find all solutions to \( \sin z = \cos z \).
   
   Hint : Let \( w = e^{iz} \).

5. (a) Find all values of \( (1 + i)^i \).

(b) What are the accumulation points of the values of \( (1 + i)^i \)?

(c) Let \( z \) be on the unit disk. That is, \( |z| = 1 \). If \( c \) is any real number, what can you say about the values of \( z^c \)?

(d) Use the definition of log to find and simplify all values of \( \log e^z \).

(e) For any \( z = x + iy \), find the unique value of \( \log e^z \).
   
   Hint : First try to find \( \log e^{8i} \).

6. Give precise statements of the following :

(a) The Cauchy-Goursat Theorem

(b) The Cauchy Integral Formula for Derivatives

(c) The Maximum Modulus Principle

7. Let \( C \) be the contour which consists of the line segment from \( z = -i \) to \( z = 1 \).

Evaluate \( \int_C xy \, dz \).

8. Let \( C \) denote the boundary positively oriented rhombus with corners \( 1, 2i, -1, -2i \), traversed once.

For any \( w \) not on \( C \), define \( g(w) = \int_C \frac{z^2e^z}{(z - w)^2} \, dz \).

(a) Evaluate and simplify \( g \left( \frac{\pi i}{2} \right) \), and justify your answer.

(b) Evaluate and simplify \( g(2 + 2i) \), and justify your answer.
9. Let \( f(z) \) be analytic on the closed, bounded set \( R \). Suppose that \( f(z) \neq 2 \) for any \( z \in R \).

**True or False:** The minimum value of \( |f(z) - 2| \) for \( z \in R \) is achieved on the boundary of \( R \).

Explain your answer.

10. Consider the function \( f(z) = \frac{1}{(z - 3 + i)(z + 1 + i)} \).

Write the domains for all Laurent expansions of \( f(z) \) centred at \( z = 0 \).

11. Find the Laurent series expansion with centre \( z = 0 \) for \( f(z) = \frac{1}{1 - z} - \frac{1}{2 + i - z} \) which is valid in an annulus containing \( z = 2 \).

Write your series in the form \( f(z) = \sum_{n=0}^{\infty} a_n z^n + \sum_{n=1}^{\infty} b_n z^{-n} \).

12. Consider the Laurent series expansion \( f(z) = \sum_{n=0}^{\infty} \left( \frac{1}{n!} + 5^n \right) \frac{1}{z^{2n}} \).

(a) Write a functional expression for \( f(z) \) which does not involve series.

(b) Find the region in which this series expansion is valid.

13. Recall that \( \sin z = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n+1)!} = \frac{z}{1!} - \frac{z^3}{3!} + \frac{z^5}{5!} - \cdots \) for all \( z \).

Let \( f(z) = \sin \frac{1}{z} \).

(a) Find a Laurent expansion for \( f(z) \)

(b) Find the domain in which this series converges.

(c) Determine the type of singularity of \( f(z) \) at \( z = 0 \).

(d) For which integers \( n \) is the residue of \( z^n f(z) \) non-zero?

(e) Let \( C \) be a simple closed curve in the positive orientation whose interior contains \( z = 0 \).

Evaluate \( \int_C z^{20} f(z) \, dz \).

14. (a) Write a function \( g(z) \) which is analytic, except for a simple pole at \( z = i \), a pole of order 2 at \( z = 1 \), and an essential singularity at \( z = 0 \).

(b) Find the residue of \( g(z) \) at \( z = i \).

15. By multiplying the appropriate series together, find the residue of \( f(z) = \frac{e^{1/z}}{1 - z} \) at \( z = 0 \), and express the answer in terms of well-known constants.

16. Evaluate \( \int_{|z| = 2} \frac{\sin z}{z^2(z + 1 + i)} \, dz \).