1. Suppose that we wish to fit a data set \( \{(x_i, y_i)\}_{i=1}^n \) (where all values are positive), with a power function of the form \( y = ax^b \).

We can choose a least squares fit in the form

\[
\text{minimize: } f(a, b) = \sum_{i=1}^{n} \left( \ln y_i - \ln a - b \ln x_i \right)^2
\]

or

\[
\text{minimize: } g(a, b) = \sum_{i=1}^{n} (y_i - ax_i^b)^2
\]

(a) Solve the first problem analytically.

(b) For each fixed \( b \), \( g(a, b) \) is a quadratic function of \( a \). Set \( a \) to the minimum value of this quadratic, and convert the problem to a 1-dimensional minimization.

(c) Suppose that \( \hat{a} \) and \( \hat{b} \) solve the first problem, and let

\[
\delta_i = \ln y_i - \ln \hat{a} - \hat{b} \ln x_i, \quad 1 \leq i \leq n.
\]

Write \( g(\hat{a}, \hat{b}) \) in terms of these values. This gives an upper bound on the minimum value. Can you determine if this value is larger, or smaller than \( f(\hat{a}, \hat{b}) \)?

2. Consider Rosenbrock’s Banana Function: \( f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2 \), which has a unique minimum at \((1, 1)\).

(a) Find and simplify the gradient \( \nabla f(x) \) and Hessian \( \nabla^2 f(x) \).

(b) Describe the region for which \( \nabla^2 f(x) \) is positive definite.

(c) The point \((1, 2)\) does not lie in this region. Show that Newton’s method converges in 1 step with this starting point.

(d) The point \((1.1, 1)\) does lie in this region. If we start at this point with Newton’s method, show that we move away from the minimum.