1. A class is given the problem of finding a point on the intersection of the surfaces \( f(x) = 0 \) and \( g(x) = 0 \).
   
   (a) A student suggests minimizing the function \( h(x) = (f(x) - g(x))^2 \)
   
   Explain the problem with this idea.
   
   (b) Suggest a modification which will work, assuming that the surfaces do intersect.

2. Consider the objective function \( f(x_1, x_2) = (x_1 + x_2 - 6)^2 - 10 \tan^{-1}((2x_1 + 3x_2 - 5)^8) \)
   
   (a) Show that \( f \) is bounded below.
   
   (b) Find any critical points of \( f \).
   
   (c) Show that \( f \) has no global minimum. What does this say about minimizing the function?
   
   (d) Given the initial point \( x_0 = (20, -11) \) and descent direction \( p = \langle -2, 1 \rangle \), construct and simplify the line search function \( g(\alpha) = f(x_0 + \alpha p) \), for \( \alpha \geq 0 \).
   
   (e) Show that the choice \( \alpha = 0.5 \) satisfies the strong Wolfe conditions for the above line search function, with \( c_1 = 0.0001 \) and \( c_2 = 0.9 \).

3. Consider the linear system

\[
\begin{bmatrix}
3 & -1 & 0 & 0 \\
-1 & 3 & -1 & 0 \\
0 & -1 & 3 & -1 \\
0 & 0 & -1 & 3
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix}
= 
\begin{bmatrix}
5 \\
4 \\
-3 \\
0
\end{bmatrix}
\]

(a) Without calculating determinants, show that the eigenvalues of the coefficient matrix \( A \) are real and positive.

   **Hint** Look up Gershgorin’s Theorem.

(b) Use *exact* arithmetic and conjugate gradient minimization with starting vector \( x_0 = 0 \) to solve this linear system.