1. Write $\frac{5 - 3i}{(2 + i)(1 - i)}$ in the form $a + ib$.

2. Use the definition of $e^z$ to prove that $|e^z| = e^x$.

3. Use polar coordinates to find the exact value of $(1 - i\sqrt{3})^{30}$ in the form $a + ib$.

4. Write the 4th roots of $-16$ in the form $a + ib$. 


5. Explain the difference between:

“$f(z)$ is differentiable at $z = z_0$” and “$f(z)$ is analytic at $z = z_0$”

5 points

6. Find $f'(z)$, if $f(z) = \frac{\exp(z^3 + 5z)}{z^4 + 1}$.

10 points

7. Use Euler’s formula to derive the trigonometric expansions of $\cos(\theta + \phi)$ and $\sin(\theta + \phi)$.

10 points

8. Use the limit definition to show that $f(z) = \Xi$ is not differentiable at $z = 0$.

5 points
9. Let \( v = xy + e^{-x} \cos y \).
   a) Show that \( v \) is harmonic.

b) Find a function \( u \) such that \( u + iv \) is analytic.

10. Consider the function \( f(z) = \frac{z - i}{z + i} \).
   a) Prove that, if \( z \) is real, \( |f(z)| = 1 \).

b) By the above, \( f(z) \) maps the real axis onto the unit circle \(|z| = 1\). What does this function do to the upper-half plane \( \text{Im } z \geq 0 \)?