1. Write the following in the form $a + bi$.

   (a) \[ \frac{2 - i}{(1 + 2i)(1 - 3i)}. \]

   (b) \[ (-2\sqrt{3} + 2i)^5 \]

2. (a) Show that the mapping $w = \frac{z}{1 + z}$ transforms the region to the right of $\text{Re } z = -\frac{1}{2}$ to the open disk $|w| < 1$. 

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(b) **Recall** that the mapping \( w = \frac{1}{z} \) transforms the region \( |z| < 1 \) to \( |w| > 1 \).

Use this to sketch the image of the set \( |z - (-1)| < 1 \) under the mapping \( w = 2 + \frac{3}{1 + z} \).

3. Let \( v(x, y) = 3x^2 + 8xy - 3y^2 + 4x - 5y \).

   (a) Show that \( v \) is harmonic everywhere.

   (b) Find the harmonic conjugate \( u(x, y) \).
4. Let \( f(z) = \frac{1}{1 + 2i - z} \).

(a) Find and simplify the power series expansion of \( f(z) \) centred at \( z = 2 \), valid near this point.

(b) Determine the radius of convergence of this series.

5. Graduate Students: Find the radius of convergence of the power series \( \sum_{n=1}^{\infty} \frac{z^n!}{2^n} \).