1. A certain computer system requires a password of between 7 and 10 characters. Each character can be a letter (upper and lower-case are equivalent), or a digit. The password must be typed in the correct order.

   (a) If there is no other restriction, how many different passwords are possible?

   (b) If there is a further restriction that at least one character is a digit, how many passwords with exactly 10 characters are there?

   (c) Suppose that we know that Pam’s password consists of the characters ”a,b,j,d,1,2,5,6”, in some order. If we try different orders at random, and the system locks us out after 5 tries, what is the probability that we will gain access?

2. Sugary Sam’s Donut House has 14 different kinds of doughnuts available.

   (a) In how many ways can we pick 1 dozen (12) doughnuts, if they are all different?

   (b) In how many ways can we pick 1 dozen (12) doughnuts, if there is no restriction at all?

   (c) In how many ways can we pick 2 dozen (24) doughnuts, if we must choose at least one of each kind?

3. (a) Given a class of 9 people, in how many ways can we divide them into groups of size 4,3,2 for a project?
   Restate this as an occupancy problem (balls in cells).
(b) **Graduate Students Only**: Given a class of 9 people, in how many ways can we divide them into groups of size 3,3,3?
   Restate this problem as an occupancy problem (balls in cells).

4. Evaluate the sum \( \sum_{k=1}^{n} \binom{n}{k} \frac{k(k-2)}{2} \) by differentiating the expansion of \((1+\sqrt{x})^n\) twice, and setting \(x = 1\).

5. Describe the kind of graph-theoretic model which you would use to model the following, noting the differences between them:
   (a) A sewer system for a particular street.
   (b) The telephone network for a local exchange.

6. Four married couples are at a banquet, and are to be seated at a circular table.
   (a) If spouses sit next to one another, in how many different ways could we seat them?
   (b) If spouses sit next to one another, and Mr. Jones cannot sit next to Mrs. Smith or Mrs. Abrams, describe a graph-theoretic model that could be used to construct a seating chart.
   (c) **Graduate Students Only**: How many possible seating arrangements are possible in the previous case?