1. Given the functional equation \( x^2y^3z^4 - e^{xz} + y^2 = (xy + z)^2 - 1 \), use implicit differentiation to find simplified forms for \( z_x, z_y \).

**Solution:** We write \( F(x, y, z) = x^2y^3z^4 - e^{xz} + y^2 - (xy + z)^2 + 1 = 0 \).

Then,
\[
F_x = 2xy^3z^4 - ze^{xz} - 2y(xy + z),
\]
\[
F_y = 3x^2y^2z^4 + 2y - 2x(xy + z),
\]
and
\[
F_z = 4x^2y^3z^3 - xe^{xz} - 2(xy + z).
\]

So,
\[
z_x = -\frac{F_x}{F_z} = \frac{-2xy^3 + ze^{xz} + 2y(xy + z)}{4x^2y^3z^3 - xe^{xz} - 2(xy + z)}
\]
and
\[
z_y = -\frac{F_y}{F_z} = \frac{-3x^2y^2z^4 - 2y + 2x(xy + z)}{4x^2y^3z^3 - xe^{xz} - 2(xy + z)}.
\]

2. Find the point(s) on the elliptic paraboloid \( z = x^2 + 3y^2 \) such that the tangent plane cuts the \( x \)-axis at \( x = 3 \) and the \( y \)-axis at \( y = 2 \).

**Solution:** Let \( z = f(x, y) = x^2 + 3y^2 \).

Then, the tangent plane is
\[
f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) - (z - z_0) = 0,
\]
where \( f_x(x_0, y_0) = 2x_0, f_y(x_0, y_0) = 6y_0 \) and \( z_0 = x_0^2 + 3y_0^2 \).

So, we have
\[
2x_0(x - x_0) + 6y_0(y - y_0) - (z - x_0^2 - 3y_0^2) = 0.
\]

Since the plane passes through \((3,0,0)\):
\[
2x_0(3 - x_0) = 6y_0(0 - y_0) - (0 - x_0^2 - 3y_0^2) = 0,
\]
which simplifies to:
\[
(1) \quad 6x_0 - x_0^2 - 3y_0^2 = 0.
\]

The plane also passes through \((0,2,0)\):
\[
2x_0(0 - x_0) + 6y_0(2 - y_0) - (0 - x_0^2 - 3y_0^2) = 0,
\]
which simplifies to:
\[
(2) \quad - x_0^2 + 12y_0 - 3y_0^2 = 0.
\]

Looking at (1) and (2), we have
\[
(3) \quad x_0 = 2y_0.
\]
Substitute this into (1):

\[ 12y_0 - 4y_0^2 - 3y_0^2 = 0, \]

so that \( y_0 = 0 \) or \( y_0 = 12/7 \).

From this, the points are \((0, 0, 0)\) and \(\left(\frac{24}{7}, \frac{12}{7}, \frac{144}{7}\right)\).

3. Suppose that \( f(1, 1) = e - 1 \), and

\[
\frac{\partial f}{\partial x} = -\cos(x^2) - 4x^3
\]

and

\[
\frac{\partial f}{\partial y} = \cos(y^2) + 2ye^{y^2}.
\]

Use differentials to estimate \( f(1.01, 0.98) \)

**Solution:** We have

\[
f(1.01, 0.98) \approx f(1, 1) + f_x(1, 1)(0.01) + f_y(1, 1)(-0.02)
\]

\[= e - 1 + (-\cos 1)(0.01) + (\cos 1 + 2e)(-0.02) \approx 1.5533\]

4. Let \( G(s, t) = F(u(s, t), \ v(s, t)) \) where \( F, u, v \) are differentiable.

Given \( u(1, 0) = 2, \ u_s(1, 0) = -2, \ u_t(1, 0) = 6, \ v(1, 0) = 3, \ v_s(1, 0) = 5, \ v_t(1, 0) = 4, \)

\( F_u(2, 3) = -1, \ F_v(2, 3) = 10. \)

Find, if possible, the values of \( G_s(1, 0) \) and \( G_t(1, 0) \).

**Solution:** We have

\[
G_s(1, 0) = F_u(u(1, 0), v(1, 0)) \cdot u_s(1, 0) + F_v(u(1, 0), v(1, 0)) \cdot v_s(1, 0)
\]

\[= F_u(2, 3) \cdot u_s(1, 0) + F_v(2, 3) \cdot v_s(1, 0) = (-1)(-2) + (10)(5) = 52\]

and

\[
G_t(1, 0) = F_u(u(1, 0), v(1, 0)) \cdot u_t(1, 0) + F_v(u(1, 0), v(1, 0)) \cdot v_t(1, 0)
\]

\[= F_u(2, 3) \cdot u_t(1, 0) + F_v(2, 3) \cdot v_t(1, 0) = (-1)(6) + (10)(4) = 34.\]