1. Determine the number of intervals required to use the Trapezoidal Rule to estimate \[ \int_{1}^{2} \sin(x^2) \, dx \] to 6 decimal places (error less than \(5 \times 10^{-7}\)).

**Answer:** It suffices that \(n \geq 1733\).

**Hint:** With \(f(x) = \sin(x^2)\), we have \(f''(x) = 2 \cos(x^2) - 4x^2 \sin(x^2)\). A simple estimate for \(1 \leq x \leq 2\) gives \(|f''(x)| \leq 18\), with a more precise value of \(|f''(x)| \leq 10.802\). The first value gives \(n \geq 1733\), the better value gives \(n \geq 1342\).

2. Repeat the previous question using Simpson’s Rule.

**Answer:** It suffices that \(n \geq 48\).

**Hint:** We have \(f''''(x) = -12 \sin(x^2) - 48x^2 \cos(x^2) + 16x^4 \sin(x^2)\). For \(1 \leq x \leq 2\), a simple bound is 460, while the more precise estimate is 165. The first value gives \(n \geq 48\), and the second one gives \(n \geq 37\).

3. Determine which of the following integrals are improper. Explain your answers.

(a) \(\int_{1}^{2} \frac{dx}{2x-1}\)
(b) \(\int_{0}^{1} \frac{dx}{2x-1}\)
(c) \(\int_{-\infty}^{\infty} \frac{\sin(x)}{1 + x^2} \, dx\)
(d) \(\int_{1}^{2} \ln(x-1) \, dx\)

**Answer:** The second is improper because the integrand is discontinuous at \(x = 1/2\), inside the interval.
The third is improper because of the infinite limits.
The fourth is improper, since the integrand is improper at \(x = 1\).

4. If possible, give the exact value of the improper integral \(\int_{0}^{\infty} \frac{e^x}{4e^{2x} + 9} \, dx\).

**Answer:** The value is \(\frac{1}{6} \left(\frac{\pi}{2} - \tan^{-1}\left(\frac{2}{3}\right)\right)\).

**Hint:** First try \(u = 2e^x\).

5. If possible, give the exact value of the improper integral \(\int_{0}^{1} \ln(x) \, dx\)

**Answer:** The value is -1.

**Hint:** Use l’Hospital’s Rule to evaluate \(\lim_{a \to 0^+} a \ln(a)\).