1. An 1100-gal tank initially contains 200 gal of a chlorine-water mixture containing 40 lb of chlorine. Water containing 9 lb of chlorine per gallon enters the tank at a rate of 4 gal/s, and the perfectly mixed solution in the tank flows out at a rate of 1 gal/s. How much chlorine will the tank contain when it is full?

Solution: The net flow into the tank is 3 gal/s, which means that the volume of fluid after \( t \) seconds is \( 200 + 3t \), and the tank fills in 300 s.

Let \( C(t) \) denote the amount of chlorine at time \( t \). Since the concentration at that time is \( \frac{C(t)}{200 + 3t} \), we obtain the linear, first-order equation

\[
C'(t) = 36 - \frac{C(t)}{200 + 3t}
\]

or

\[
C'(t) + \frac{1}{200 + 3t} C(t) = 36.
\]

Using the integrating factor \( \exp \left( \int \frac{dt}{200 + 3t} \right) = \exp \left( \frac{1}{3} \ln(200 + 3t) \right) = (200 + 3t)^{1/3} \), this becomes

\[
\frac{d}{dt} \left( (200 + 3t)^{1/3} C(t) \right) = (200 + 3t)^{1/3} C'(t) + (200 + 3t)^{-2/3} C(t) = 36(200 + 3t)^{1/3}
\]

Integrating both sides,

\[
(200 + 3t)^{1/3} C(t) = 9(200 + 3t)^{4/3} + A.
\]

Using the initial condition \( C(0) = 40 \) yields \( A = -1760(200)^{1/3}, \) and so \( C(300) = 9(200 + 3(300))^1 - 1760(200)^{1/3}(200 + 3(300))^{-1/3}, \) which is approximately 8903 lb.

2. A 19-lb turkey, initially at 35°F, is put into a 325°F oven at 6:00 A.M.; it is found that the temperature \( T(t) \) of the turkey is 60°F after 75 min. When will the turkey be 160°F?

Solution: Let the temperature of the turkey after \( t \) minutes be \( T(t) \). Then we have Newton’s Law of Cooling

\[
T'(t) = k(325 - T)
\]

Integrating, and observing that \( T(t) < 325 \) yields

\[
- \ln(325 - T) = kt + A
\]

Putting in the initial condition \( T(0) = 35 \) yields \( A = -\ln(290) \). Putting in the condition \( T(75) = 60 \) yields \( k = \frac{\ln(290/265)}{75}. \)

Solving for \( T(t) = 160 \) gives \( t = 75 \frac{\ln(290/165)}{\ln(290/265)}, \) which is approximately 469 minutes. The desired time is thus about 1:49 pm.
3. The initial mass of a certain species of fish is 3 million tons. The mass of fish, if left alone, would increase at a rate proportional to the mass, with a proportionality constant of $\frac{7}{3}$ /year. However, commercial fishing removes fish mass at a constant rate of 8 million tons per year. When will all the fish be gone? If the fishing rate is changed so that the mass of fish remains constant, what should that rate be?

**Solution:** Let $M(t)$ be the mass of fish at time $t$ in millions of tons. We then have the simple model

$$M'(t) = \frac{7}{3}M - 8,$$

which can easily be solved with the initial condition $M(0) = 3$ to give

$$M(t) = \frac{3}{7} \left(8 - e^{\frac{7}{3}t}\right)$$

a) $M(t) = 0$ when $e^{\frac{7}{3}t} = 8$, or approximately 11 months (0.89 years).

b) If we set the harvest rate to keep the same population level, then it should be 7 million tons per year.

4. A parachutist whose mass is 120 kg drops from a helicopter hovering 5500 m above the ground and falls toward the earth under the influence of gravity. Assume that the gravitational force is constant and that the force due to air resistance is proportional to the velocity of the parachutist, with the proportionality constant $k_1 = 40$ kg/s when the chute is closed and with constant $k_2 = 140$ kg/s when the chute is open. If the chute does not open until 1.5 min after the parachutist leaves the helicopter, after how many seconds will he hit the ground?

**Solution:** Let $x(t)$ denote the height above ground after $t$ seconds. Notice that $x'(t) \leq 0$, since he is falling. We first consider $0 \leq t \leq 90$, with the ’chute closed. By Newton’s Second Law, we have

$$120x'' = -40x' - 120g$$

where $g = 9.81 \text{ m/s}^2$ is the acceleration due to gravity. Integrating once,

$$120x' = -40x - 120gt + A$$

Putting in the initial conditions $x(0) = 5500, x'(0) = 0$ yields $A = 40(5500)$. We write the equation in standard linear form as

$$x' + \frac{1}{3}x = \frac{A}{120} - gt$$

and use the integrating factor $\exp\left(\frac{1}{3}dt\right) = \exp\left(\frac{1}{3}t\right)$ to obtain

$$x(t) = \frac{A}{40} + 9g - 3gt + Be^{-\frac{t}{3}}$$

Again using the initial conditions gives $B = -9g$, and so we have

$$x(t) = 5500 - 3gt + 9g \left(1 - e^{-\frac{t}{3}}\right)\quad\text{and}\quad x'(t) = -3g \left(1 - e^{-\frac{t}{3}}\right).$$

When the ’chute opens after 90 seconds (using $g = 9.81 \text{ m/s}^2$), $x(90) \approx 2939$ m and $x'(90) \approx -29.43$ m/s.
The terminal speed with the 'chute open is \( v_T = \frac{120g}{140} \), which is slower than the man falling. Hence, when the 'chute opens, he is “instantly” slowed to speed \( v_T \). Hence, the total time of descent is \( 90 + \frac{x(90)}{v_T} \approx 439.6 \) seconds.

Alternatively, we can solve the problem the instant that the 'chute opens, which has equation

\[ 120x'' = -140x' - 120g. \]

Matching the values of \( x(90) \) and \( x'(90) \) yields

\[ x(t) \approx -\frac{6}{7}gt - 8.3865400678 \times 10^{46}e^{-7t/6} + 3678.343061 \]

Solving for \( x(t) = 0 \) yields \( t \approx 437.45 \) seconds.