Consider the function  \( f(x) = x^4 \left( 2 + \sin \left( \frac{1}{x} \right) \right) \).

Clearly, \( f(x) \geq 0 \) for all \( x \), and it has a global minimum at \( x = 0 \).

For integers \( n \), with \( |n| \) sufficiently large, \( f(x) \) also has a local minimum close to \( x_n = \frac{1}{2\pi n - \frac{\pi}{2}} \).

By writing this minimum as \( x = \frac{1}{2\pi n - \frac{\pi}{2} + \epsilon} \), and using Taylor’s Theorem, estimate the location of these local minima.

5 points