1. Determine whether the existence and uniqueness theorem does or does not guarantee existence of a solution of the given initial value problem. If existence is guaranteed, determine whether the theorem does or does not guarantee uniqueness of that solution:

(a) \( \frac{dy}{dx} = \sqrt{\sqrt{y} - e^x} \), \( y(1) = e^2 \).

(b) \( \frac{dy}{dx} = \sqrt{\sqrt{y} - e^x} \), \( y(-1) = e^{-1} \).

2. Find the general solution to \( xe^y \frac{dy}{dx} = (36x^2 \sin x^2 - 12x^2 e^y \sin x^2) + (6 - 2e^y) \).

3. Solve the initial value problem \( \frac{dy}{dx} + xy^2 e^{-x} = -xe^{-x}, y > 0 \), subject to \( y(0) = 3 \).

4. Solve the initial value problem \( \left[-3x^2 y^2 \sin (x^3 y^2) + y^3 e^{xy^3} - 18x^{1/2} y^{1/3}\right] dx = \left[2x^3 y \sin (x^3 y^2) - 3x^2 e^{xy^3} + 4x^{3/2} y^{-2/3}\right] dy \), subject to \( y(1) = 0 \).