Homework #1  Analytic Function Theory 3450:625  Dr. Norfolk

1. Suppose that one value of $z^{1/3} = 3 - 5i$. In the form $a + ib$, find the exact values of

   (a) $z$

   (b) The other 2 values of $z^{1/3}$.

2. Let $P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0$ be a polynomial of actual degree $n \geq 2$ ($a_n \neq 0$) with real coefficients.
   
   If $z$ is a non-real zero of $P(x)$, prove that $P(x)$ is divisible by $x^2 - (2\text{Re } z) x + |z|^2$.

3. Write the values of $(3 + 2i)^{1/2}$ in the form $a + ib$ without trigonometric functions.

4. Suppose that $|z| = 1$ and $z \neq -1$.

   Show that $2 \text{Arg}(1 + z) = \text{Arg}(z)$. 