1. Let \( r(z) = \frac{p(z)}{q(z)} \) be a non-degenerate rational function. That is, \( p(z) \) and \( q(z) \) are polynomials with no common factors, and \( r(z) \) does not reduce to a constant.

For each complex number \( w \), find the number of solutions to \( r(z) = w \), in terms of the degrees of \( p, q \) and their coefficients.

2. Find the radius of convergence for the Taylor series to \( f(z) = \frac{1}{\sin(\log z)} \), about the point \( z_0 = 2 + i \).

3. Let \( \theta \) be real. Use Taylor series to find closed forms for the sums \( \sum_{n=0}^{\infty} \frac{\cos n\theta}{n!} \) and \( \sum_{n=1}^{\infty} \frac{\sin n\theta}{n!} \).

4. Find the annulus of convergence for the Laurent series \( \sum_{n=1}^{\infty} \left(1 - \sin \left(\frac{1}{3n}\right) \right)^n z^n + \sum_{n=1}^{\infty} \frac{(\ln n)^{\sqrt{n}}}{z^n} \).

5. Determine where the function \( f(z) = \sum_{n=0}^{\infty} \frac{(2n)!}{2^{2n}(n!)^2} \left( \frac{z+1}{z-1} \right)^n \) is analytic.

6. Prove, or find a counterexample : If \( \sum_{n=0}^{\infty} a_n z^n \) represents an entire function, then \( \sum_{n=0}^{\infty} a_n z^{n^2} \) is also entire.

7. Use the Residue Theorem to evaluate \( \int_0^{2\pi} \frac{e^{i\theta}}{2 - e^{-i\theta}} \, d\theta \).

Separate the real and imaginary parts to deduce the values of the associated trigonometric integrals.

8. Suppose that \( f(z) \) is a non-polynomial entire function, and the equation \( f(z) = z^4 - 1 \) has simple roots at \( z = \pm 1, \pm i \).

Prove that \( f(z) \) has an infinite number of zeros, using Picard’s Little Theorem.