1. Use Laplace transforms to solve the differential equation

\[ y(0) = 1, \ y' + 6y = \begin{cases} 
0, & \text{if } 0 \leq t < 3 \\
e^{-6t}, & \text{if } 3 \leq t 
\end{cases} \]

2. Use Laplace transforms to solve the differential equation

\[ y'' - 4y' + 4y = 56t^6e^{2t}, \ y(0) = 0, \ y'(0) = 1 \]
3. Use Laplace transforms to find $y(t)$ ONLY, where $x(0) = 0$, $y(0) = 0$ and

\[
x' - 2x - y = 2 \\
x + y' - 2y = 0
\]

4. Use power series about $x = 0$ to find a general solution to $y'' - 2xy' + x^2y = 0$.

Write your solution in the standard form $y = Af(x) + Bg(x)$, where $f(x)$ and $g(x)$ are power series up to the terms in $x^6$. 

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5. Find an explicit solution to $\sqrt{x} \frac{dy}{dx} - 2e^y = 0$, $y(1) = 0$

6. Find the general solution to the differential equation $x \frac{dy}{dx} + (6 + 2x)y = 48x^{18}e^{-2x}$. 

7. Find the general solution to the equation $x^2 y'' + 4xy' + 2y = e^x$.

8. Find the general solution to $y'' - 4y' + 3y = e^x + 3x$. 

15 points

15 points

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9. Find the general solution to \[ \begin{align*} x' &= x + 3y \\ y' &= -3x + y. \end{align*} \]

10. Given the differential equation \( y'' + 5y' + 6y = f(t), y(0) = a, y'(0) = b, \) the Laplace transform \( Y(s) = \mathcal{L}\{y\} = \frac{s - 1}{(s - 2)^2(s + 2)}. \)

Find \( f(t), a \) and \( b. \)