1. Use Laplace transforms to solve the differential equation
\[ y' + 3y = g(t), \text{ where } y(0) = 2, g(t) = \begin{cases} 0, & 0 \leq t < 5 \\ 4e^{-3t}, & 5 \leq t \end{cases}. \]

15 points

2. Use Laplace transforms to solve the differential equation
\[ y'' + 6y' + 9y = 20t^3e^{-3t}, \ y(0) = 0, \ y'(0) = 1. \]

15 points
3. Use Laplace transforms to find $x(t)$ ONLY, where $x(0) = 0$, $x'(0) = 0$, $y(0) = 0$, $y'(0) = 0$

\[ x' - 7x + 2y' = t \]
\[ x' + 2x - y'' = 0 \]

4. Use power series about $x = 0$ to find a general solution to

\[ y'' - 2xy' + 3y = 0 \]
5. Solve \( \sqrt{x} \frac{dy}{dx} - e^{-\sqrt{x}} y^5 = 0 \), \( y(0) = \frac{1}{2} \).

6. Solve the differential equation \( x \frac{dy}{dx} + (4 + 3x)y = 32x^{11} e^{-3x} \).
7. Find the general solution to the equation \( x^2 y'' + 4xy' + 2y = \sqrt{x+2} \).

8. Find the general solution to \( y'' - 16y = te^{2t} + 16t - 48 \).
9. The eigenvalues of \[
\begin{pmatrix}
-4 & 1 \\
-4 & -8
\end{pmatrix}
\] are \( \lambda = -6, -6 \).

Find the general solution to
\[
\begin{align*}
x' &= -4x + y \\
y' &= -4x - 8y 
\end{align*}
\]