1. Define the following terms:

(a) The *Cartesian product* of sets $A$ and $B$

(b) $R$ is a *relation* from $A$ to $B$

(c) The *domain* of the relation $R$ from $A$ to $B$

(d) The *range* of the relation $R$ from $A$ to $B$

(e) The *composition* of the relations $R$ and $S$ (i.e. $S \circ R$)

(f) $R$ is a *reflexive* relation on $A$

(g) $R$ is a *symmetric* relation on $A
(h) $R$ is an *antisymmetric* relation on $A$

(i) $R$ is a *transitive* relation on $A$

(j) $R$ is an *equivalence relation* on $A$

(k) The *equivalence class* of $x$ under the equivalence relation $R$ on $A$

(l) $R$ is a *partial order* on $A$

(m) $f$ is a *function* from $A$ to $B$

(n) $f$ is *one-to-one* (injective) from $A$ to $B$

(o) $f$ is *onto* (surjective) from $A$ to $B
2. Let \( P \) be the set of all currently living people. Define a relation \( C \) on \( P \) via 
\[ x \ C \ y \text{ if and only if } x \text{ is a biological child of } y. \]
In English, precisely describe the following:

(a) The domain of \( C \).

(b) The range of \( C \).

(c) The composition \( C \circ C \).

3. Let \( T = \{1, 2, 3, \ldots, 999\} \) be the set of all natural numbers less than 1000.
Define an equivalence relation \( S \) on \( T \) via \( m \ S \ n \) if and only if the sum of the digits of \( m \) and \( n \) is the same.
For example, \( 72 \ S 333 \), since \( 7 + 2 = 3 + 3 + 3 \)

(a) List all members of the equivalence class \( 2 \ / \ S \).

(b) List all members of the equivalence class \( 21 \ / \ S \).

(c) How many equivalence classes are there?
4. Let \( V = \{1, 2, 3, \cdots, 20\} \). Define a relation \( D \) on \( V \) via
\[ p \, D \, q \text{ if and only if } p \text{ exactly divides } q. \]

(a) *Prove* that \( D \) is a *partial order*.

(b) Explain whether or not \( D \) is a *linear/total order*.

5. Define \( g : \mathbb{N} \to \mathbb{N} \) via \( g(1) = 5, \ g(3) = 11, \) and \( g(n + 2) = 2g(n + 1) - g(n) \) for all \( n \geq 1 \).

*Prove* that \( g(n) = 3n + 2 \) for all \( n \geq 1 \).
6. Let $A = \{1, 2, 3, 4\}$. Define $s : \mathcal{P}(A) \to \{0, 1, 2, \cdots, 10\}$, via $s(B)$ is the sum of the members of $B \subseteq A$. (Note: $\mathcal{P}(A)$ is the power set of $A$, and $s(\emptyset) = 0$).

e.g. $s(\{1, 2, 4\}) = 1 + 2 + 4 = 7$.

(a) Prove or disprove: $s$ is one-to-one.

(b) Prove or disprove: $s$ is onto.

(c) Write $s^{-1}(4)$ explicitly.