1. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}$ is the vector field

$$\mathbf{F}(x, y, z) = \sin(\pi x)\mathbf{i} + \cos(\pi y)\mathbf{j} + xz\mathbf{k}$$

and $C$ is the curve $\mathbf{r}(t) = t^3\mathbf{i} - t^2\mathbf{j} + t\mathbf{k}$, $0 \leq t \leq 1$

2. Show that $\mathbf{F}(x, y, z) = (2xy - \cos x)i + (1 + x^2 + e^y)j$ is a conservative field, by finding an associated potential function.
3. Evaluate $\int_C \nabla f \cdot dr$, where $f(x, y, z) = \tan(e^{xy+z})$ and $C$ is the curve 
\[ r(t) = t^4i - \cos t j + \frac{1}{\pi} \sin^{-1} t k, \quad 0 \leq t \leq \frac{1}{2}. \]

4. Evaluate $\int_C -y^3dx + x^3dy$ where $C$ is the boundary of the region bounded by $y = \sqrt{9 - x^2}$ and $y = 0$, traversed counterclockwise.
5. Evaluate \( \text{curl } \mathbf{F} = \nabla \times \mathbf{F} \) and \( \text{div } \mathbf{F} = \nabla \cdot \mathbf{F} \), where \( \mathbf{F}(x, y, z) \) is the vector field
\[
\mathbf{F}(x, y, z) = (x^2 - 2y)i + (x + 2z)^2j + (x - y^2)k.
\]

6. Evaluate the flux of the vector field \( \mathbf{F}(x, y, z) = (x^2 + y^2)i + xyj - yzk \) through the surface \( S \) of the region in the first octant bounded by \( x^2 + y^2 = 4 \) and \( x + 2y + 3z = 6 \).
7. Suppose that $f(x, y, z)$ is twice continuously differentiable, and that $x = 4u + 3v - w$, $y = u + 2v + 3w$, $z = u + v - 2w$, Find a simplified expression in terms of $f_{xx}, f_{yy}, f_{zz}, f_{xy}, f_{yz}, f_{zx}$ for $\frac{\partial^2 f}{\partial u \partial w}$.

8. Consider a tetrahedron, with vertices at $A(0,0,0)$, $B(2,0,0)$, $C(0,3,0)$ and $D(1,1,4)$. Find the angle that the line through the origin and the barycenter (average coordinate) of face BCD makes with the normal to BCD.
9. An object moves on the curve \( \mathbf{r}(t) = \cos t \mathbf{i} + \sin t \mathbf{j} + t^2 \mathbf{k} \). for \( t \geq 0 \). Find \( v, a_T, a_N \) when \( t = \sqrt{2} \) (distances in metres, time in seconds).

10. Find the minimum value of \( x^2 + 2y^2 + 3z^2 \) on the plane \( x + 2y - 3z = 3 \)