Name ____________________________

Final Examination  Dec 10, 2007  Honors Calculus I 3450:221  Drs. Clemons/Norfolk
Show all of your work, and simplify your answers. Give exact answers where possible.

1. Find $f'(x)$, where $f(x) = \sqrt{(4x^2 + 1)^3} \div (2x - 3)^{1/4}$.

2 points

2. Find $y''$, where $y = \sin^2(4x)$.

6 points

3. A solid rubber cylinder has length 200 mm and radius 2 mm. It is stretched at 10 mm/sec, in such a way that it remains the same shape, and same volume. Find the rate of change of the radius when the cord is 800 mm long.

8 points
4. Given \( g'(x) = \frac{3(1-4x)}{x^4} \), \( g''(x) = \frac{12(3x-1)}{x^5} \), \( \lim_{x \to 0^-} g(x) = +\infty \), \( \lim_{x \to 0^+} g(x) = -\infty \):

(a) Find the intervals where \( g(x) \) is increasing, and where it is decreasing. Identify any local extrema.

(b) Find the intervals where \( g(x) \) is concave up, those where it is concave down, and identify any inflection points.

5. Let \( h(x) = \begin{cases} 
\frac{1}{2} - x^2 & \text{if } x < -1 \\
3 + 2x & \text{if } -1 \leq x
\end{cases} \), and find the following limits, if possible. If the limit does not exist, describe the type of discontinuity for \( h(x) \).

(a) \( \lim_{x \to -1^-} h(x) = \)

(b) \( \lim_{x \to -1^+} h(x) = \)

(c) \( \lim_{x \to -1} h(x) = \)
6. Valerie wishes to build a rectangular corral of area 900 ft$^2$ for pygmy goats, divided into 3 rectangles of equal area, using 2 dividers. If the cost of the outside fence is $10 per foot, and the cost of the dividers is $6 per foot, find the minimum possible cost.

7. Evaluate $\int \frac{x(1 - 2x) - 3}{x^{2/3}} \, dx$. 

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8. Evaluate \( \int_{0}^{1} 4x^2 \sqrt{9x^3 + 7} \, dx \).

9. Evaluate the following:

(a) \( \lim_{x \to 3^-} \frac{1 - x}{(x + 2)^2(x + 3)} = \)

(b) \( \lim_{x \to -1} \frac{5 + 4x - x^2}{2x^2 - x - 3} = \)

(c) \( \lim_{t \to 0} \frac{\sin^2(3t)}{2t \tan(4t)} = \)

(d) \( \lim_{x \to \infty} (2x - \sqrt{4x^2 - 4x + 12}) = \)
10. Evaluate \( \int_{-3/2}^{-1} 2(2x + 1) \sqrt{2x + 3} \, dx \).

11. Evaluate \( \int \left( \frac{1}{x^2} + \frac{2 \cos(x) - 6 \csc(3x) \cot(3x)}{\sqrt{\sin(x) + \csc(3x)}} \right) \, dx \).
12. **Set up, but do not evaluate** the integral(s) to find the area of the region bounded by $x + y = 4$, $y = x$ and $3y = x$.

13. Find the **average value** of $f(x) = x(3x + 2)$ on the interval $[-2, -1]$.

14. The region bounded by $y = (x - 1)^2$, $y = 0$ and $x = 3$ is rotated once about the $x$-axis, to obtain a **solid of revolution**. Find the volume of this solid, using the **method of disks**.
15. The region bounded by \( y = -1, \ x = 1, \ x = 2 \) and \( x = y^3 \) is rotated about the \( y \)-axis. **Set up, but do not evaluate** the integral(s) to evaluate the volume of the solid of revolution, using the *method of shells*.

16. The region bounded by \( x = 2y \) and \( x = y^2 \) is rotated about the line \( y = 2 \). **Set up, but do not evaluate** the integral(s) to evaluate the volume of this solid of revolution, using the *method of washers*.

17. Find the *equation* of the tangent line to \( x^3y + y^2 + y = 1 - 3x \) at the point \((-1,2)\).