1. Find all solutions to the linear system

\[
\begin{align*}
2x + 5y - \frac{9}{2}z &= 2 \\
x + 4y - 3z &= \frac{3}{2} \\
2x + 2y - 3z &= 1
\end{align*}
\]

2. Solve for \(x\):

\[
\begin{vmatrix}
x & 2 & 3 \\
0 & -x & 1 \\
5 & 0 & 1
\end{vmatrix} = 24
\]

Page 1 Total (25)
3. Use Cramer’s Rule to find a simplified solution for $x$ and $y$ in the system

\[
\begin{align*}
(c \cos \theta)x - (c \sin \theta)y &= 2 \\
(c \sin \theta)x + (c \cos \theta)y &= 3
\end{align*}
\]
(b) Let \( b_1 = 0 \), and \( b_n = n \cdot b_{n-1} + 1 \) for \( n \geq 2 \). Find the value of \( b_4 \).

(c) Suppose that \( \{c_n\} \) is an arithmetic sequence, and that \( c_3 = 6 \) and \( c_8 = 12 \). Find the value of \( c_{13} \).

(d) Suppose that \( \{d_n\} \) is a geometric sequence, and that \( d_2 = 3 \) and \( d_6 = \frac{3}{4} \). Find the common ratio \( r \).

(e) Determine if the sequence \( \frac{1}{2}, \frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \ldots \) is arithmetic, geometric, or neither.
(f) Recall that the sum of an infinite geometric series \( a + ar + ar^2 + \cdots = \frac{a}{1-r} \) if \(|r| < 1\).

Use this to write the repeating decimal \(0.23 23 23 \cdots\) as fraction.

6. For a specific positive integer \(n\), we know that
\[
1 + 2^1 + 2^2 + \cdots + 2^n = 2^{n+1} - 1 .
\]
For \(n = 1\), this becomes \(1 + 2 = 2^2 - 1\), which is true. Show that this formula works for \(n + 1\).

That is,
\[
1 + 2^1 + \cdots + 2^n + 2^{n+1} = 2^{n+2} - 1 .
\]

This proves, by the Principle of Mathematical Induction, that the formula works for every integer \(n \geq 1\).

7. Use the Binomial Theorem to find the coefficient of \(x^4\) in the expansion of \(\left(2x - \frac{3}{x}\right)^8\).