1. Let $P$ denote the set of all people alive on August 1, 2011, and $B$ be the set of all books written before that day.

Define a relation $R$ from $P$ to $B$ by $x R y$ if and only if person $x$ read book $y$ before that same day.

Precisely describe the domain and range of $R$ in words.

10 points

2. Describe a relation on $A = \{1, 2, 3, 4\}$ which is reflexive and transitive, but not symmetric. Give the relation as a set of ordered pairs, and as a graph.

10 points

3. Let $P$ be the set of all living people. Define a relation $K$ on $P$ by $x K y$ if and only if $x$ and $y$ have the same biological father or mother.

Prove or disprove:

(a) $K$ is reflexive

5 points

(b) $K$ is symmetric

5 points

(c) $K$ is transitive

5 points

4. Let $Z = \{10, 11, 12, 13, \ldots, 99\}$ be the set of 2-digit decimal integers. Define the equivalence relation $F$ on $Z$ via $ab F cd$ if and only if $a + 2b = c + 2d$.

(a) List the elements of 3 different equivalence classes.
(b) How many equivalence classes are there?

5. Define a function $s$ with domain $\mathbb{N}$ by a pair, whose first element is the smallest prime that divides the number, and the second is the number of primes which divide it.
For example, $s(12) = (2, 2)$, since $2 \mid 12$ and the prime factors of 12 are 2 and 3.
Note: 1 is not prime.
Describe an appropriate codomain for $s$.

6. Define $f : \mathbb{N} \to \mathbb{Q}$ by $f(1) = 1$, $f(2) = 2$ and $f(n) = \frac{f(n-1) + f(n-2)}{2}$ for $n \geq 3$.
Prove that $f(n) \leq 2$ for all $n$.

7. Define $f : \mathbb{R} \to \mathbb{R}$ via $f(x) = 2\lceil x \rceil - x$.
(a) If $n \in \mathbb{Z}$, find $f(n)$.
(b) If $n \in \mathbb{Z}$, find $f(n + 0.5)$.
(c) If $n \in \mathbb{Z}$, find the image of the interval $(n, n + 1]$ under $f$.
(d) Show that $f^{-1}(x) = 2\lfloor x \rfloor - x$.
   Hint: Try the cases when $x \in \mathbb{Z}$, and when $x = n + \alpha$, where $n \in \mathbb{Z}$ and $0 < \alpha < 1$.  

10 points

5 points

5 points

5 points

10 points

5 points

5 points

5 points

15 points