1. Find a function \( f(x, y) \) and a region \( R \) such that
\[
\int \int_R f(x, y) \, dA = \lim_{m,n \to \infty} \sum_{i=1}^{m} \sum_{j=1}^{n} \left[ (2 + \frac{3i}{m}) \left( 1 + \frac{2j}{n} \right) - \cos^2 \left( 2 + \frac{3i}{m} \right) \right] \cdot \frac{3}{m} \cdot \frac{2}{n}
\]

2. Evaluate \( \int_{0}^{2} \int_{1}^{4} x^2 e^{-y} \, dx \, dy \)

3. Consider \( \int_{0}^{1} \int_{x}^{2x} x - y^2 \, dy \, dx \).
   (a) Evaluate this integral.
   (b) Sketch the region of integration.
(c) SET UP, BUT DO NOT EVALUATE, the problem with the order of integration reversed.

4. We may write

\[
\left( \int_{-\infty}^{\infty} e^{-x^2} \, dx \right)^2 = \int_{-\infty}^{\infty} e^{-x^2} \, dx \int_{-\infty}^{\infty} e^{-y^2} \, dy
\]

\[
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-x^2-y^2} \, dA = \lim_{R \to \infty} \int \int_{x^2+y^2 \leq R^2} e^{-x^2-y^2} \, dA
\]

By evaluating the last quantity using polar coordinates, find the value of \( \int_{-\infty}^{\infty} e^{-x^2} \, dx \).

5. A lamina is bounded by the circle \( x^2 + y^2 = 1 \), \( x, y \geq 0 \), and has density \( \rho(x, y) = x + 2y + 1 \).

WITHOUT EVALUATING THE INTEGRALS, find a formula using iterated integrals for the \( x \)-coordinate of the centre of mass.
6. WRITE, BUT DO NOT EVALUATE, an iterated integral for the surface area of the paraboloid 
\[ z = 9 - x^2 - 3y^2 \] which lies above the triangle in the \( xy \)-plane with vertices (0,0), (4,0) and 
(4,2).

7. A tetrahedron is bounded by \( x+4y+6z = 12 \), \( x, y, z \geq 0 \), and has density function \( \rho(x, y, z) = (x + y)e^{1-2z} \).

WRITE, BUT DO NOT EVALUATE, an iterated triple integral for the mass of the object.

8. The region \( D \) is bounded by the cylinder \( x^2 + y^2 = 9 \), \( z = 0 \), \( z = 4 \), \( y = 0 \), \( y = x \), \( x \geq 0 \).

REWRITE, BUT DO NOT EVALUATE a simplified integral in cylindrical coordinates for 
\[ \int \int \int_D x^2 dV. \]
9. The region $D$ is bounded by the sphere $x^2 + y^2 + z^2 = 11$, $y, z \geq 0$. REWRITE, BUT DO NOT EVALUATE a simplified iterated integral in spherical coordinates for $\int \int \int_D x^2 + z^2 dV$

10. Consider the transformation $x = u - v, y = uv$.

   (a) Draw the region in the $uv$-plane which corresponds to the rectangle $0 \leq x \leq 1, 1 \leq y \leq 3$.

   (b) Use the Jacobian to find $dxdy$ in terms of $dudv$. 