1. Mark each of the following statements True or False, as appropriate:
   (a) $\emptyset \in \{a, b\}$  __________________________
   (b) $\emptyset \subseteq \{a, b\}$  __________________________
   (c) $|\{a, \{b, c\}\}| = 2$  __________________________
   (d) If $A, B, C$ are sets, then $A \cap (B \cup C) = (A \cap B) \cup (B \cap C)$  __________________________
   (e) For any set $A$, $A \cap \emptyset = A$  __________________________

2. Use Venn diagrams to show that $(A - C) \cap (B - C) = (A \cap B) - C$.

3. Consider the following claim for non-empty sets $A, B, C$:
   If $A \cap B = \emptyset$ and $B \cap C = \emptyset$, then $A \cap C = \emptyset$.
   Find a counterexample to this statement.
4. Consider the following true statement:
For any elements $x, y, z$ in a Boolean algebra: \((x'\cdot y)\cdot z + x = x + (y\cdot z)\)

(a) Fill in the justification for the following steps of the proof:
\[
((x'\cdot y)\cdot z + x = \\
\quad = x + ((x'\cdot y)\cdot z) \\
\quad = x + (x'\cdot (y\cdot z)) \\
\quad = (x + x')(x + (y\cdot z)) \\
\quad = 1(x + (y\cdot z)) \\
\quad = (x + (y\cdot z)) \cdot 1 \\
\quad = x + (y\cdot z)
\]

(b) Write the dual of the original statement.

(c) Write the original statement in the Boolean algebra of set theory.

5. Resolve the following statements:
Person A says "B is lying".
Person B says "A is telling the truth".

6. There are 222 balls in an urn, with consecutive odd numbers. The number on the last ball is 2321. i.e. the set is \(\{\cdots, 2317, 2319, 2321\}\). What is the number on the first ball?
7. Consider the set $H_7$ of all 7-digit hexadecimal integers.

(a) How many such integers are there?

(b) Count the integers in $H_7$ for which the first digit is not '1', and the second digit is not '2', and the third digit is not '3'.

(c) How many integers in $H_7$ contain at least one '0'? 
8. Which is more likely: To get at least one 6 in four tosses of a fair die, or to draw one card from a shuffled deck, and get either a high card (A,K,Q,J,T), or a red card? Explain your answer carefully.

5 points

9. There are 100 athletes in a triathlon, consisting of 54 adult males, 30 adult females, and 16 children.

(a) How many possible sets of 10 athletes are there?

5 points

(b) How many possible sets of 10 athletes have exactly 5 men, 3 women and 2 children?

5 points

(c) If Valerie (an adult female) and Graham (a child) must both be in the chosen set of 10 athletes, how many different final sets are possible?

5 points

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