1. Mark each of the following statements True or False, as appropriate:

(a) $\emptyset \in \{a, b\}$

(b) $\emptyset \subseteq \{a, b\}$

(c) $|\{a, \{b, c\}\}| = 2$

(d) If $A, B, C$ are sets, then $A \cap (B \cup C) = (A \cup B) \cap (A \cup C)$

(e) For any set $A$, $A \cup \emptyset = A$

2. Use Venn diagrams to show that $(A - C) \cup (B - C) = (A \cup B) - C$.

3. Consider the following claim for non-empty sets $A, B, C$:

If $A \cap B \neq \emptyset$ and $B \cap C \neq \emptyset$, then $A \cap C \neq \emptyset$.

Find a counterexample to this statement.
4. Consider the following true statement:
For any elements $x, y$ in a Boolean algebra: $(x.y).(\overline{x} + \overline{y}) = 0$

(a) Fill in the justification for the following steps of the proof:

$$
(x.y).(\overline{x} + \overline{y}) =
= ((x.y).\overline{x}) + ((x.y).\overline{y})
= ((y.x).\overline{x}) + ((x.y).\overline{y})
= (y.(x.\overline{x})) + (x.(y.\overline{y}))
= (y.0) + (x.0)
= 0 + 0
= 0
$$

(b) Write the dual of the original statement.

(c) Write the original statement in the Boolean algebra of set theory.

5. Resolve the following statements:

Person A says "B is lying".

Person B says "A is telling the truth".

6. There are 256 balls in an urn, with consecutive even numbers. The number on the first ball is 144. i.e. the set is \{144, 146, 148, \cdots\}. What is the number on the last ball?
7. Consider the set $D_7$ of all 7-digit decimal integers.

(a) How many such integers are there?

(b) How many of these integers do not have '1' as the first digit, '2' as the second digit, \ldots , '7' as the seventh digit?

(c) How many integers in $D_7$ have at least one '0'?
8. Which is more likely: To get \textit{at least} 1 Head in three tosses of a fair coin, or to draw one card from a shuffled deck, and get either a high card (A,K,Q,J,T), or a red card? Explain your answer carefully.

9. There are 30 dogs in a Dog Show, consisting of 17 males and 13 females.
   (a) How many possible sets of the top 6 dogs are there?

   (b) How many possible sets of the top 6 dogs have at least 2 females?

   (c) If Whimsy and Cleo must \textit{both} be in the final set of 6 dogs, how many different final sets are possible?