Show all of your work and explain your reasoning.

1. Either evaluate the following limits, or show that they do not exist:

   (a) \( \lim_{(x,y) \to (-1,3)} \frac{(x+1)^2-(y-3)^3}{(x+1)^2+(y-3)^2} \).

   5 points

   (b) \( \lim_{(x,y) \to (0,0)} \frac{x^3y^3}{x^2+y^2} \).

   5 points

2. Find all of the first partial derivatives of: \( u = y^{3/2} \)

   15 points

3. Find \( \frac{\partial^2 f}{\partial y^2} \) for \( f(x,y) = \int_0^{x-y^2} \sin t^4 \, dt \).

   10 points
4. Show that \( u = e^{-kx} \cos(ky) \), where \( k \) is constant, satisfies the \textit{Laplace’s equation} 
\[ u_{xx} + u_{yy} = 0. \]

5. Find the \textit{linearization} of \( f(x, y) = \cos(x + \sin(y)) \) at the point \( \left( \frac{\pi}{2}, 0 \right) \).

Use it to estimate the value of \( f(1.56, 0.2) \)

6. Let \( z = F(x, y), x = g(s, t), y = h(s, t) \). Find \( z_s \) when \( s = 1, t = 2 \), given that \( F_x(1, 2) = 2, F_y(1, 2) = -1, F_x(3, 0) = 6, F_y(3, 0) = 2, g(1, 2) = 3, h(1, 2) = 0, x_s(1, 2) = -1 \) and \( y_s(1, 2) = 5 \).

7. Find \( D_u f \) for the function \( f(x, y) = e^{(x-1)y^3} \) at the point \( (1, 1) \) in the \textit{direction of} \( \mathbf{v} = -3\mathbf{i} + 4\mathbf{j} \).
8. Let \( f(x, y) = xy^2 + \frac{1}{x} + \frac{1}{y} \).

Show that the critical points are given by \( \left( \sqrt{2}, \frac{1}{\sqrt{2}} \right) \) and \( \left( -\sqrt{2}, -\frac{1}{\sqrt{2}} \right) \) and classify them using the Second Derivative Test.
9. Use \textit{Lagrange Multipliers} to find the extremum of $x^2 + y^2 + 2z^2$ on the plane $2x + 6y + 10z = 45$. Explain whether this point is a maximum or a minimum.