1. Either evaluate the following limits, or show that they do not exist:

(a) \( \lim_{(x,y) \to (2,3)} \frac{(x - 2)(y - 3)}{(x - 2)^2 + (y - 3)^2} \).

5 points

(b) \( \lim_{(x,y) \to (0,0)} \frac{x^2 y}{x^2 + y^2} \).

5 points

2. Find all of the first partial derivatives of:

(a) \( u = \frac{x^y}{z} \).

15 points

(b) \( f(s, t) = \frac{st^2}{s^2 + t^2} \).

10 points
3. Show that $u = \sin(kx)\sin(akt)$, where $a, k$ are constants, satisfies the wave equation $u_{tt} = a^2 u_{xx}$.

4. Find the linearization of $f(x, y) = \sqrt{x + e^y}$ at the point $(3,0)$.
   Use it to estimate the value of $f(2.9, 0.2)$

5. Given that $z = f(x, y, t), x = g(t), y = h(t)$, use the Chain Rule to find an expression for $\frac{dz}{dt}$.

6. Find $D_uf$ for the function $f(x, y) = \tan^{-1}(xy^2)$ at the point $(1,1)$ in the direction of $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$.
7. Find all critical points of \( f(x, y) = (1 + xy)(x + y) \), and classify them using the Second Derivative Test.
8. Use Lagrange Multipliers to find the maximum and minimum of $2x + 6y + 10z$ on the surface $x^2 + y^2 + z^2 = 35$. 

15 points