1. Consider the statement "Addition of integers is commutative".
   (a) Write this statement as a symbolic statement with quantifiers.
   
   5 points

   (b) Write the symbolic negation of this statement.
   
   5 points

2. (a) Find the values of $-1937 \div 16$ and $-1937 \mod 16$.

   5 points

   (b) Directly prove the following:
   
   $(\forall a, b \in \mathbb{Z})(a \mod 7 = 2 \land b \mod 7 = 5 \rightarrow 7 \mid a + b)$

   10 points
3. Use the Euclidean algorithm to evaluate \( \gcd(82861, 62197) \).

4. (a) For real numbers \( x, y \), consider the two statements:
(A) \( \lfloor x \rfloor + \lceil y \rceil \geq \lfloor x + y \rfloor \)
(B) \( \lfloor x \rfloor + \lceil y \rceil \leq \lceil x + y \rceil \).
Which of these statements is always true? (A), (B), both, or neither? Carefully explain your answer.

(b) Find all real \( x \) such that \( \lceil 0.3 - 2x \rceil = -2 \).

5. Prove by contradiction that \( 3 - 5\sqrt{2} \) is irrational.
6. (a) Completely state (symbolically), the *Principle of Mathematical Induction* (PMI)

(b) Use the PMI to prove that $1 + 2 + \cdots + 2^n = 2^{n+1} - 1$ for every positive integer $n$. 
7. (a) Find an explicit formula which generates the sequence
\[
\frac{2}{1}, \frac{5}{2}, \frac{8}{4}, \frac{11}{8}, \frac{14}{16}, \ldots
\]
(b) Write \(1 - 8 + 27 - 64 + 125 - 216\) in *summation form*.
(c) Evaluate and simplify the product
\[
\prod_{j=1}^{6} \frac{j+3}{j+1}
\]
(d) Suppose that \(\prod_{k=1}^{38} a_k = \frac{335}{112}\), and that \(a_{39} = 5\).

Find the value of \(\prod_{k=1}^{39} a_k\).
(e) Using the summation formula \(\sum_{k=1}^{n} 6k^2 = n(n + 1)(2n + 1)\), find the value of \(\sum_{k=12}^{37} 6k^2\).