1. Consider the statement "The integers are closed under division".
   (a) Write this statement as a symbolic statement with quantifiers.

   (b) Write the symbolic *negation* of this statement.

2. (a) Find the values of $-1737 \div 15$ and $-1737 \mod 15$.

   (b) Directly prove the following:
   $$(\forall a, b \in \mathbb{Z})(a \mod 3 = 1 \land b \mod 3 = 2 \rightarrow 3 \mid a + b)$$
3. Use the Euclidean algorithm to evaluate \( \text{gcd}(88725, 31977) \).

4. (a) For real numbers \( x, y \), consider the two statements:
   (A) \( \lfloor x \rfloor + \lceil y \rceil \geq x + y \)
   (B) \( \lfloor x \rfloor + \lceil y \rceil \leq x + y \).
   Which of these statements is always true? (A), (B), both, or neither? Carefully explain your answer.

(b) Find all real \( x \) such that \( \lceil 3 - 2x \rceil = 5 \).

5. Prove by contradiction that \( 2 + 3\sqrt{2} \) is irrational.
6. (a) Completely state (symbolically), the *Principle of Mathematical Induction* (PMI)

(b) Use the PMI to prove that \(1 + 3 + \cdots (2n - 1) = n^2\) for every positive integer \(n\).
7. (a) Find an explicit formula which generates the sequence \( \frac{2}{1}, \frac{4}{2}, \frac{8}{6}, \frac{16}{24}, \frac{32}{120}, \ldots \).

(b) Write \( 1 - 4 + 9 - 16 + 25 - 36 \) in **summation form**.

(c) Evaluate and simplify the product \( \prod_{j=1}^{6} \frac{j}{j + 2} \).

(d) Suppose that \( \prod_{k=1}^{200} a_k = \frac{22}{7} \), and that \( a_{200} = 5 \). Find the value of \( \prod_{k=1}^{199} a_k \).

(e) Using the **summation formula** \( \sum_{k=1}^{n} 4k^3 = n^2(n + 1)^2 \), find the value of \( \sum_{k=12}^{37} 4k^3 \).