1. Solve the initial value problem explicitly:

\[ \frac{dy}{dx} = \frac{\cot(x)\sqrt{1-y^2}}{\cos(x)}, \quad y \left(\frac{\pi}{2}\right) = 1 \]

2. Given the Bernoulli equation \( x \frac{dy}{dx} - 3y = \frac{\sqrt{x}}{\sqrt{y}} \), use an appropriate transformation to transform it to a linear equation in standard form.

DO NOT SOLVE THE EQUATION
3. Solve the linear equation explicitly
\[(5 - 2x) \frac{dy}{dx} - 4y = \frac{1}{(x + 1)(x + 3)}, \quad y(0) = 0.\]

4. Consider the equation
\[\left(\frac{2x}{y^2} - \frac{4y^2}{x^3} + g(y)\right) \, dx - \left(\frac{2x^2}{y^3} - \frac{4y}{x^2} + x \sin(y)\right) \, dy = 0.\]

(a) Find the function \(g(y)\) so that this equation is exact.

(b) Use this value of \(g(y)\), and solve the equation.
5. A particle of mass 2 kg is moved from rest at the origin on the \(x\)-axis by a force of \(1 - e^{-t}\) Newtons to the right (where \(t\) is in seconds). There is friction equal to the speed.

(a) Write a differential equation, with appropriate initial condition, to describe the velocity.

\[5 \text{ points}\]

(b) Assuming that you wrote the correct equation, we can solve this for the velocity \(v(t)\):

\[v(t) = (1 - e^{-t/2})^2 = 1 - 2e^{-t/2} + e^{-t}\]

Find the exact time when the velocity is \(1/4 \text{ m/s}\)

\[5 \text{ points}\]

(c) What happens to the velocity as \(t \rightarrow \infty\)?

\[3 \text{ points}\]

(d) Find the position function \(s(t)\)

\[7 \text{ points}\]
6. Use an appropriate substitution to find the general solution to
\[
\frac{dy}{dx} = \frac{y^2 + x^2 e^{-y/x}}{xy}
\]

7. Use the substitution \( u = \cos(2x) - y \) to find the general solution explicitly to
\[
\frac{dy}{dx} = -2 \sin(2x) + \cot(\cos(2x) - y).
\]