1. Solve the initial value problem

\[ \frac{dy}{dx} = y \tan^2(3x), \ y(0) = 2. \]

2. Given the Bernoulli equation \( \frac{dy}{dx} + x^2 y = x^2 \sqrt{y} \), use an appropriate transformation to transform it to a linear equation in standard form.

DO NOT SOLVE THE EQUATION
3. Solve the linear equation
\[ x \frac{dy}{dx} - 3y = x^4 e^{4x}, \ y(1) = 0. \]

4. Consider the equation
\[ \left( \frac{2x}{y^2} - 4xy + 3y^2 - ay \right) \ dx - \left( \frac{2x^2}{y^3} + 2x^2 - 6xy + 4x - 5 \right) \ dy = 0. \]

(a) Find the value of \( a \) so that this equation is exact.

(b) Use this value of \( a \), and solve the equation.
5. A particle of mass 2 kg is moved from rest on the x-axis by a force of 10 Newtons to the right. There is friction proportional to the speed, with constant 6 kg/s.

(a) Write a differential equation, with appropriate initial condition, to describe the velocity.

(b) Assuming that you wrote the correct equation, and that the particle starts at the origin, the solution is

\[ v = \frac{5}{3} \left(1 - e^{-3t}\right) \], \[ s = \frac{5}{9} \left(3t + e^{-3t} - 1\right). \]

Find the exact position of the particle when the acceleration is 4 m/s².

6. A population of a species naturally grows subject to the equation \( \frac{dP}{dt} = 0.2 \ P \). Suppose that we start with 1000 individuals, and harvest at a rate \( \frac{P}{t + 5} \).

How many will we have when \( t = 5 \)?
7. Use an appropriate substitution to find the general solution to

\[ \frac{dy}{dx} = \frac{y \tan \left( \frac{y}{x} \right) + 2x \sin \left( \frac{y}{x} \right)}{x \tan \left( \frac{y}{x} \right)}. \]

8. Use a substitution to find the general solution to

\[ \frac{dy}{dx} = 3e^{2x-y+4} + 2. \]