1. Construct the complete truth table for the propositional form

\[ P \lor Q \Rightarrow Q \land \neg R \]

2. Find the values (if possible) of \( P \) and \( Q \) for which the following are false:

(a) \([(P \Rightarrow Q) \Rightarrow P] \Rightarrow P\)

(b) \( P \iff P \land (P \lor Q)\)
3. Find *counterexamples* to the following statements:

   (a) \((\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(y^2 = x^3)\)

   (b) \((\forall a, b, c \in \mathbb{Z})(a \mid b \land a \mid c \Rightarrow b \mid c)\)

   (c) \((\forall a, b, c \in \mathbb{N})(a \mid c \land b \mid c \Rightarrow ab \mid c)\)

4. (a) Find a simplified *contrapositive* to the statement:
   \((\forall x \in \mathbb{R})(P(x) \Rightarrow \neg Q(x) \land R(x))\)

   (b) Find a simplified *denial* to the statement: \((\exists n \in \mathbb{N})(P(n) \Rightarrow \neg Q(n))\)
5. Consider the true statement:
"There is no smallest positive real number"
This may be expressed as:
"For any positive real number, there exists a smaller positive real number"

(a) Write this as a quantified statement with domain \( \mathbb{R}^+ = (0, \infty) \).

(b) Prove this statement.

6. Grade the following "proofs".
If the grade is 'A', describe the basic method of proof.
If the grade is 'C', describe the error(s), and how to correct them.
If the grade is 'F', describe the error(s).

(a) **Claim.** For any set \( A \), \( A \cap \emptyset = A \)

"**Proof.**" We know that \( x \in A \cap \emptyset \) iff \( x \in A \) and \( x \in \emptyset \). Since \( x \in \emptyset \) is false, \( x \in A \) and \( x \in \emptyset \) iff \( x \in A \). Therefore, \( x \in A \cap \emptyset \) iff \( x \in A \); that is, \( A \cap \emptyset = A \). 

(b) **Claim.** For an irrational number \( t \), \( 5t \) is also irrational.

"**Proof.**" Suppose that \( 5t \) is rational. Then, there exist integers \( p, q \) with \( q \neq 0 \) such that \( 5t = \frac{p}{q} \). Hence, we have \( t = \frac{p}{5q} \), where \( p \) and \( 5q \) are integers, and \( 5q \neq 0 \). That is, \( t \) is rational. Hence, \( t \) irrational implies \( 5t \) irrational.
7. For any set $A$, let $\wp(A)$ denote the \textit{power set} of $A$.

   Explain which of the following are true for $A \neq \emptyset$, and why.

   (a) $A \in \wp(A)$.

   (b) $A \subseteq \wp(A)$.

5 points

8. Let

   $E$ be the set of \textit{even} integers,
   $O$ be the set of \textit{odd} integers,
   $P$ be the set of \textit{positive} integers,
   $N$ be the set of \textit{negative} integers.

   Find simplified expressions for:

   (a) $P - N$
   (b) $P - O$
   (c) $P \cup N$
   (d) $E \cap N$

10 points

9. \textit{Prove}: Given any two sets $A, B$, we have $A \cap B \subseteq A$. 

10 points