1. Construct the complete *truth table* for the *propositional form*: \( P \land Q \Rightarrow Q \lor \sim R \)

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
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<td>T</td>
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</tbody>
</table>

10 points

2. Suppose that \( P \Rightarrow Q \) is *false*. Evaluate:

(a) \((P \Rightarrow \sim Q) \Rightarrow Q\)

5 points

(b) \(P \land Q \iff \sim P \lor Q\)

5 points

(c) \(\sim (P \lor Q)\)

5 points

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3. (a) Write a simplified contrapositive of \((\forall \epsilon \in \mathbb{R}) \left( \epsilon > 0 \Rightarrow (\exists n \in \mathbb{N}) \left( \frac{1}{n} < \epsilon \right) \right)\).

(b) Write a simplified denial of \((\forall x \in \mathbb{R}) (x \leq 1 \Rightarrow (\exists y \in \mathbb{R}) (\sin y = x))\).

4. Find counterexamples to the following statements:

(a) \((\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(y^2 \leq x^3)\)

(b) \((\forall a, b, c \in \mathbb{Z})(a \mid b \land b \mid c \Rightarrow b \mid a + c)\)
5. Proofs:

(a) Give a direct (Modus Ponens) proof of \( (\forall p, q \in \mathbb{Z})(p \mod 5 = 1 \land q \mod 5 = 4 \Rightarrow 5 \mid p+q) \)
(Recall that \( n \mod d \) is the remainder when \( n \) is divided by \( d \))

(b) Give an indirect (Modus Tollens) proof of \( (\forall m \in \mathbb{Z})(7m - 5 \text{ even } \Rightarrow m \text{ odd }) \)
(You may assume that \( \sim (m \text{ odd}) \iff m \text{ even } \).)
(c) Give a proof by contradiction of \((\forall x \in \mathbb{R})(x \notin \mathbb{Q} \Rightarrow 2 + 3x \notin \mathbb{Q})\)

6. (a) Suppose that \(a\) is an integer, and \(a \mod 17 = 5\).
Find the value of \((12a) \mod 17\).

(b) Use the Euclidean GCD Algorithm to find GCD(5445,3003).
7. Grade the following "proofs".
If the grade is 'A', describe the basic method of proof.
If the grade is 'C', describe the error(s), and how to correct them.
If the grade is 'F', describe the error(s).

(a) Claim. If \( p \) is prime, then \( p + 73 \) is composite.

"Proof." Let \( p \) be any prime. If \( p = 2, \) then \( p + 73 = 75 = 3(25), \) which is composite. If \( p \neq 2, \) then \( p \) is odd, so \( p + 73 \) is even, and hence composite. •

(b) Claim. If \( x \) is irrational and \( r \) is rational, then \( rx \) is irrational.

"Proof." Suppose that \( x \) is irrational, \( r \) is rational, and \( rx \) is rational. Then, we can find integers \( p, q, \) with \( q \neq 0, \) such that \( rx = \frac{p}{q}. \) But, \( \mathbb{Q} \) is closed under division, so that \( x = \frac{p}{qr} \) is also rational. This contradicts our assumption that \( x \) is irrational, so the statement holds. •