

Name: _____

Quiz 19, Section 7.4, due on Thu 3 July(10 pts) Use Laplace Transforms to solve the initial value problem $y' - y = te^t \sin t$, $y(0) = 0$.

The right side is a double modification of $\sin t$. The e^t indicates a shift, and the t indicates a negative derivative. It goes thusly:

$$\begin{aligned}\mathcal{L}(\sin t) &= \frac{1}{s^2 + 1} \\ \mathcal{L}(t \sin t) &= -F'(s) = -\frac{-2s}{(s^2 + 1)^2} = \frac{2s}{(s^2 + 1)^2} \\ \mathcal{L}(te^t \sin t) &= \frac{2(s-1)}{((s-1)^2 + 1)^2}\end{aligned}$$

Hence you get

$$\begin{aligned}(s-1)Y(s) &= \frac{2(s-1)}{((s-1)^2 + 1)^2} \\ Y(s) &= \frac{2}{((s-1)^2 + 1)^2}\end{aligned}$$

This is a shifted version of #25 with $k = 1$, so you get the result

$$y(t) = (\sin t - t \cos t)e^t$$